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International Textbooks in Civil Engineering

A HISTORICAL APPRAISAL OF MECHANICS

IT HAS BEEN SAID that no man is civilized or mentally adult until he realizes that the past, the present, and the future are indivisible.

A Historical Appraisal of MECHANICS

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Foreword

All branches of engineering rest upon a foundation of mechanics. To appreciate the present and to plan for the future, one must have an understanding of the past. It is to be regretted that so little attention has been given in the literature of engineering to the historical aspects of mechanics, too few writers realizing that a study of the past has a bearing on the present and future. In this book, the author has endeavored to include factual historical material which should prove helpful to students and workers in technology who are interested in the evolution of mechanics.

Mechanics is an old branch of learning. It is traced in this treatise to the days when the Greeks dominated all fields of learning, stressing the speculative and the accepted. Aristotle and Archimedes made significant contributions to mechanics of ancient times, the Bacons in the Medieval period, and Leonardo da Vinci and Copernicus during the Renaissance. Galileo, the creator of modern science, Descartes, Huygens, and Isaac Newton played a most important role in changing emphasis from the speculative to the factual, from the accepted to the analytical, and from the deductive type of reasoning to the inductive type. Experimentation and analysis found outlets in the study of the mechanics of materials and led to a practical theory of elasticity. Then followed graphical statics, photo-elasticity, theory of redundant structures, and other new concepts, leading to modern engineering mechanics.

The major engineering societies—particularly the American Society of Mechanical Engineers, through its Applied Mechanics Division—and several of the leading American manufacturers are now focusing the attention of engineers upon the importance of mechanics in connection with the solution of practical problems. While mechanics has at all times been considered a basic subject in engineering educa-

tion, recent years have witnessed special attention by engineering colleges to graduate and research programs in this important branch of learning.

Engineers and scientists are indebted to Professor Harvey Frank Girvin for an able and accurate presentation of the history of mechanics and of its relation to engineering education.

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Lafayette, Indiana
November, 1947

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The Creation of Science and Scientific Thinking

CHAPTER 1

Introduction

Interest in History of Science.—The story of the development and accomplishments of general science is probably less well known than any other part of the history of man. Historians expend vast quantities of words and energy filling innumerable volumes in order that they may tell us about economics, politics, and war. But not much space has been devoted to the activities and accomplishments of the men who have brought us out of our rock caves into our modern homes and have revolutionized our philosophy of thought about natural phenomena.

The reasons for this apparent indifference to scientific growth by historians are probably two-fold. Science, as we think of it today, is still a stripling when the period of its development is compared to the life-span of our social and political development; also, science has been the child of a specially trained group, while our social and political development is everybody's problem.

It is only since the beginning of the present century that any important interest in the history of science has developed. The first authoritative periodical devoted to this field was the Isis, first published in Belgium in 1913. Soon after, the History of Science Society, an international group with headquarters in America, was organized.

That the study of history by scientists and engineers is gaining momentum is indicated by the following quotation from the Journal of Engineering Education for March, 1940, in a report of a special committee: ". . . understanding of the evolution of the social organization within which we live and of the influence of science and engineering on its development can be obtained only through some type of historical

study. The ability to recognize and to make a critical analysis of a problem involving social and economic elements, to arrive at an intelligent opinion about it and to read with discrimination and purpose toward these ends . . ." Such understanding and ability can grow only through study in the field of science. Likewise, since the public is also beginning to suspect that science is a basic or fundamental way of life, much more comprehensive and complicated than anything formerly known, it will be wise for one to learn something of the background of science in order that the developments of the future shall be more understandable.

The fundamental concepts of physical science are now recognized to be abstractions which we have developed in our minds to bring some sort of order and clarity into what the casual observer thinks of as the haphazard development and confusion of nature. Men of science no longer believe, as they believed in Aristotle's time and throughout the Middle Ages, that they are dealing or working at any given period with the ultimate in physical conceptions. They now approach their problems with open minds, unhampered by restrictions of religious creed or political dogma.

The Scope of Mechanics.—Sir William Dampier, Fellow of Trinity College, Cambridge, defines physical science as "an ordered knowledge of the natural phenomena and the rational study of the relations between the concepts in which those phenomena are expressed." It develops out of man's desire to observe and study ordinary natural phenomena as they occur around him and out of his effort to construct tools and mechanical implements to improve his physical comfort.

Just what part of this activity constitutes the Science of Mechanics? There are many definitions to be found in the literature—some good and others entirely unsuitable to our present-day conception of the science. Concise definitions

that have been given are as follows: Professor Irving P. Church (1851–1931) of Cornell University stated that "Mechanics treats of the mutual actions and relative motions of material bodies, solid, liquid and gaseous"; Professor W. J. M. Rankine (1820–72) of the University of Glasgow defined Mechanics as the "Science of rest, motion and force"; and Professor Ernst Mach (1838–1916) of the University of Vienna defined it as that branch of science which is "concerned with the motions and equilibrium of masses."

Examination of these definitions of Mechanics immediately brings up the question of just what we mean by matter, motion, force and mass. An etymological study of the words does not help greatly, since we find that these terms spring from words which are commonly used to describe muscular activities or the results of such activity. For example, we learn that "Weight is the resistance to lifting," that "Mass is an agglutinated lump of kneaded dough," and that "Force is that which produces results such as follow from muscular exertion."

Such definitions would limit the Science of Mechanics to dealing with the results of muscular activity alone, and would hardly satisfy our modern conception. But, when we remember that these terms came into use during a period when muscular activity was the chief source of power known to mankind, their etymological meaning is logical enough.

Since the fundamental terminology of the science is based upon such elementary concepts, we immediately realize that we must go back a long way in our effort to trace its origin and that we shall be handicapped considerably by the incompleteness of the records.

History of Science Parallels the Development of Mankind.—Mechanics is a science which has developed primarily through the efforts of man to improve his physical well-being

rather than his cultural and esthetic state. The history of the science practically parallels the historic development of mankind. The geologists tell us that the earth's crust is at least 25,000,000 years old. There is also good evidence that the earth has been a dwelling place for some sort of human creatures for at least 25,000 years. Anthropologists believe that the valleys of the Nile, Euphrates, Ganges, and Hoang-Ho rivers were inhabited that long ago.

It has been suggested that some of the crude implements found, which are thought to be 20,000 years old, might have been the work of some super type of ape or similar species. We have the statement of Darwin (1809–1882) in *The Descent of Man* that "I have myself seen a young orang put a stick into a crevice, slip his hand to the other end, and use it in the proper manner as a lever." Many similar instances could be quoted, but there is no evidence that any animal other than man ever has been able to fabricate tools or implements for special purposes. No animal possesses the type of brain to do the creative work required for the production of even the simplest tools and implements. The fabrication of the crudest of stone implements required great skill and a considerable amount of experience and patience, if the chipping of the stones into any desired form was to be successfully accomplished. This has been clearly demonstrated when modern man has attempted to duplicate the efforts of the ancients under approximately the same conditions.

Man Begins to Create and Plan.—Just when man learned to make and control fire is not definitely known, but the evidence available seems to indicate that the date also was about 20,000 years ago. This was an exceedingly important step in his development, but it took thousands of years before man learned to make any practical use of this powerful tool.

The next important advance came with the tilling of the soil, some 8,000 to 12,000 years ago. The harvesting of food naturally led to stabilization of habitation and was a stimulus also to the manufacture of tools. However, the development of the more primitive tools was not the result of a scientific approach to the problem at hand, but was simply a process of evolution. Examination of the early Egyptian and Assyrian monuments clearly demonstrates that these ancient people had great mechanical skills and many implements, but they left no records of formulated scientific knowledge except in isolated cases; one brilliant exception was an early description of the propagation of sound as similar to the waves caused by throwing stones into the water. But for each such example there are many which show errors so absurd that they could not have been made if any important scientific culture had existed.

When and where science actually began is difficult to determine. There is no evidence of any first steps in the Science of Mechanics in the Far East. The people of the yellow and black races were just as backward scientifically in the early ages as they are today. All the early developments took place along the shores of the Mediterranean.

Why Science Was Developed.—The birth of science—or possibly we should say the birth of the scientific method of investigation—probably occurred when the volume of man's instinctively developed facts and experiences began to be unmanageably large. His primitive discoveries may not have furnished the impetus for further thought and investigation. However, as the body of knowledge grew, and certain groups of people became eminent for their skills, there was felt the need of formulating their knowledge so that it might be passed on to the younger members of the particular group and the group's social advantage and prestige might thereby be maintained.

If this was to be accomplished, it was necessary that the knowledge be studied, classified, and put in a form that could be conveniently recorded. This, then, is the essence of the scientific approach: to discover the basic principles involved and to formulate them into a readily usable pattern, so that, when certain phenomena occur, it will be possible to predict what will next develop. This is the true business of science.

When man began to collect and classify knowledge about any particular phenomenon, two questions soon developed in his mind: "*Why* did this happen?" and "*How* did it happen?" But from the scientific viewpoint it is not so important to know "why something happens" as to know "how it happens." When the early scientific investigators stopped trying to find some divine reason for things happening and devoted all their energies to discovering "how it happened," they began to make real progress in the field of science. As soon as they began to collect and classify their observations in terms of cause and effect, they observed that a definite sequence of events always produced the same result. And, as they became familiar with these repeated sequences, they were able to formulate the fundamental laws of nature in accordance with which such sequences occurred.

This, then, was the beginning of the scientific inductive process. It resulted in the development of the Science of Natural Philosophy, of which Mechanics is one of the most important branches or subdivisions.

Of this process of sifting and formulating experience, T. H. Huxley (1825–1895) said in *Science and Education:* "Science is nothing but organized common sense." That is, the scientist plans and organizes his work and therefore maintains a control over the results, while the generally haphazard methods of the untrained individual are likely to produce unexpected results or consequences. It was Herbert Spencer (1820–1903) who stated in *First Principles* that "On the application of rational mechanics depends the success

of nearly all modern manufacture." The truth of this statement is today even more evident than in Spencer's time.

Crude Arithmetic Appears.—After the evolutionary development of crude tools and such simple mechanical devices as levers, oars, pulleys, and windlasses had progressed sufficiently to make it advantageous for man to remain in one place, the need arose for some means of measuring (crude surveying) and of keeping accounts. The Babylonians and Phoenicians developed some skill in this direction, and the Phoenicians had, besides, some knowledge of astronomy. Geometry seems to have gotten its start in consequence of the seasonal overflowing of the Nile. After the waters subsided, it was always necessary to resurvey the land so that taxes could be properly levied. The Egyptians are credited also with the development—about 2500 B.C.—of a crude arithmetic whose characters were pictures of frogs, men, houses, and other objects.

At about this same time man began to use copper extensively for tools and implements. Copper was in general use on the island of Cyprus about 3,000 B.C. It was soon discovered (2500 B.C.) that the addition of tin to copper resulted in a superior metal for tools. Thus we had the first alloy. How this alloy was developed is not known. But, since there are few ores containing both metals, it is doubtful that the discovery was accidental.

The next step in man's mastery of his physical environment was the control of water. As we leave prehistoric Egypt and come to the beginning of written records (4000–3000 B.C.), we find irrigation already well developed. Then followed the period of the construction of the Pyramids, the most triumphant accomplishment of man the builder until the twentieth century with its towering steel-girdered skyscrapers. Exactly how these great stone structures were built is anybody's guess, but it was not with the aid of any

important mechanical devices. Certainly they are remarkable evidences of simple engineering skills, organization and planning. The quarrying, transporting, and erecting of the giant obelisks at Thebes and elsewhere along the Nile give further evidence of the amazing mechanical ability of those early Egyptians. Some of those stones weigh 500 tons each. The super-obelisk of all is a giant block of granite estimated to weigh 1168 tons, which still lies unfinished in the quarries at Asswan. A bad fissure suggests that the break in the stone, rather than its great weight, prevented the Egyptians from transporting it the 100 miles from Asswan to Thebes. The erection of these giant blocks of stone was a remarkable feat for the engineers of the time; and, even though the erection of the Pyramids involved no new principles of design, the magnitude of the enterprise was tremendous. It must be remembered, however, that the Egyptians had the advantage of unlimited slave labor and time; and that cost, the constant *bête noir* of present-day engineers, was entirely absent from their problem.

Before attempting to trace the story of Mechanics further through the various stages of a growth which has made it one of the most useful tools in the equipment of present-day engineers and scientists, it may be well to point out some of the more general facts which seem to have influenced scientific development.

One of the great lessons of history is the importance of will-power and its proper direction. It is fundamental for the nation as well as for the individual, whether leader or follower. The early Greeks failed to realize this. As a result they built up a group of scholars who were mainly interested in perpetuating their own kind and their intellectual development as professional students. They were not interested in applying the fruits of this development or at least part of it to the improvement of the nation and the individuals who constituted the nation.

It is difficult to draw a line between pure science and its applications. Sometimes the applications are discovered first and the principles are induced from them; and under other conditions the reverse is the case.

Greek Domination of Science.—With the growth of the Greek culture in Southern Europe, the special forces which have brought us to our present high state of civilization began to operate for the first time in the history of the world. The Greeks were pioneers in cultural progress; they were the founders both of science and of art. Their leaders were not afraid of the many inherited superstitions of the time, but sought and found much fundamental truth. Their speculations upon the nature of the universe were the beginning of our Natural Philosophy.

It was the Greeks who took the elementary mathematics of the Egyptians and developed them into tools which could be made to serve the physicist and the engineer. About 600 B.C. they imported geometry from Egypt and began to develop it and arithmetic into separate branches of mathematical science. During the next several hundred years Hippocrates (460–? B.C.), Aristotle (384–322 B.C.), Euclid (323–285 B.C.), and others systematized what was then known about geometry and arithmetic. Here then was the real birth of what has developed through the centuries into our modern mathematical or inductive science.

It has generally been believed that the Greeks had a distaste for any sort of manual work; and that they considered it degrading, fit only for slaves. The Greek ideal was to develop the mind; so they left the more humble occupations to others. The result was that they produced little of value in the nature of mechanical developments. They liked to speculate on anything and everything.

This notion of a complete indifference to experimentation on the part of the Greeks seems to have been exaggerated

by some writers, however, for records have been found of certain experimental work by early Greeks in sound and also in the refraction of light.

The growth of the Science of Mechanics, it shall be observed as we trace its course, is largely cyclic in nature. First we note a period of development which accompanies the discovery of some fundamental principle or principles; then, after a period of dormancy and consolidation, another period of development gets under way. Each of these active periods is largely dominated by some outstanding individual or small group, other men of ability being attracted as associates or pupils.

The first such period was that of Aristotle (384–322 B.C.) and Archimedes (287–212 B.C.). Aristotle's greatest interests and ability lay outside the field of Mechanics. He was primarily a philosopher and zoologist. His fame is due chiefly to his work in mental, moral, and political philosophy and in the sciences of zoology, physics, and meteorology. Archimedes, however, was a mathematician who was interested in the application of mathematics to physical things.

Mechanics and General Science.—Professor William Whewell (1794–1866), Master of Trinity College, Cambridge, in the introduction to his *History of the Inductive Sciences,* published in 1874, presents the following interesting comments on the historical examination of science in general:

"The completeness of historical view which belongs to such a design consists not in accumulating all the details of the cultivation of each science but in the making of the larger features of its formation. The historian must endeavor to point out how each of the important advances was made, by which sciences have reached their present position, and when and by whom each of the valuable truths was obtained. Real speculative knowledge requires the combination of two ingredients: right reason and facts to reason upon.

"Almost the whole career of the Greek schools of Philosophy, of the schoolmen of the Europe of the Middle Ages, of the Arabian and Indian philosophers, show us that we may have extreme ingenuity and subtlety, invention and connection, demonstration and method: and yet that out of these germs no physical science may be developed. We may obtain by such means logic and metaphysics and even geometry and algebra but never Mechanics and optics, chemistry and physiology.

"These sciences require a careful reference to observation and experiment. How rapid their progress may be, when they draw from such sources the material on which the mind of the philosopher employs itself, the history of those branches of knowledge for the last three hundred years abundantly teaches us."

We also quote the following statements from Robert Boyle's (1627–1691) *Works*, published in 1725, in which he attempts to show the importance of the Science of Mechanics:

"Mechanics is a science which therefore is greatly advantageous to experimental philosophy. The phenomena of this doctrine belong to the history of nature in its full extent. For instance, when a piece of wood is plunged into water and emerges and floats, even vulgar naturalists think it belongs to them to account for this phenomenon which they fancy proceeds from the positive levity of wood. May not a philosopher find out the reason why part of the floating wood keeps above the surface whilst the other sinks beneath it?

"Mechanics also helps to invent and judge of hypotheses relating to the subjects wherein they are concerned."

Boyle then lists a number of instances, such as levers, ships, rudders, and rowing boats, and says: "Whereof Mechanics will furnish us with a better account than the schoolmen and such as are ignorant of the properties of the center of gravity, several kinds of levers, the wedge, etc."

Boyle finds that many things in physics cannot be well explained nor understood without the assistance of Mechanics. Many examples of simple statics are then quoted to show that the Science of Mechanics is needed to explain them properly. He then quotes Archimedes as saying that "solids lighter than the fluids they are put in will sink till a quantity of fluid equal in bulk to the part immersed becomes equal in weight to the whole floating body."

He further states, "It would be easy to add a multitude of queries where a naturalist, ignorant in Mechanics, could give but a poor solution, which yet the Mechanics would satisfactorily answer. In proof thereof I refer the schoolmen to Aristotle's *Mechanical Problems*."

Another statement by Boyle is: "Also Mechanics will assist the naturalist to multiply experiments by the inquiries they will suggest and the inferences and applications whereto they lead." This is illustrated by calling attention to the new discoveries brought about by the Torricellian experiment and Galileo's experiments for the breaking of bodies and the force required to break them; to say nothing of the variety of useful propositions which have been deduced and still are mechanically deducible from the observation of Archimedes that "a solid body weighs less in water than in air by weight of a quantity of water equal in bulk to that body."

Boyle then goes on to say, "But methinks both mathematics and Mechanics have been too much confined to the stars, the earth, the water, and some few conspicuous parts of nature besides. Archimedes deduced hydrostatics from the application of vulgar statics to the weighing of bodies in air and water or water alone. But one considerable advantage both mathematics and Mechanics may afford to the naturalist is by schemes, figures, representations and models which greatly assist the imagination to conceive many things and by that means enable the understanding to judge them and deduce new consequences therefrom. T'would be exceed-

ingly difficult if not impossible to go through some tedious geometrical demonstrations without the help of a visible scheme."

The changing attitude of men of science is well illustrated by Professor Mach who in 1887, or 162 years later, said that science only gives information about phenomena apparent to our senses. However, few men concerned with the physical sciences in the eighteenth and nineteenth centuries were interested in philosophy, even that expressed by so able a man as Mach. They assumed that they were dealing with actual realities and that the path of scientific development had been definitely fixed; all that remained was the improvement of the technique of the investigations. With this philosophy Mach did not agree. He says, "the ultimate nature of reality is beyond the reach of our intelligence."

From this brief glimpse into the workings of the minds of a few of the great thinkers of the past, we learn that that which we today know as the inductive method or the scientific approach to investigation is really not a modern development. It is a method of thought which has been gradually developing down through the years, keeping pace with the increase in our store of knowledge. Its seeds were there in the minds of some great thinkers a long time ago. The reason for the apparently greater success of the inductive method during more recent times lies simply in its more general and widespread acceptance and use which multiplies many fold the fruits of its application.

CHAPTER 2

Early Greek Philosophy

The Speculative Method of the Greeks.—It is generally agreed that the Greeks borrowed the elements of their astronomical and mathematical learning from Egypt and the East where several of the Greek philosophers had traveled during the period from 600 to 500 B.C. These men no doubt received there the mental impetus which on their return to Europe started the Greek Schools of Philoscphy.

The Italian school led by Pythagoras (500 B.C.) was more inclined to the study of nature and its laws than to subjects of a mathematical nature, even though he too had traveled in Egypt and the East and was a great mathematician. It is Pythagoras who is credited with first proving that the square of the hypotenuse of a right-angle triangle is equal to the sum of the squares of the other two sides, the well-known Pythagorian Theorem.

Pythagoras has also another claim to fame. He was the first to assume the title of Philosopher. This was a name used to designate men who spent their lives in the quest for knowledge for the love of knowledge rather than for hope of any pecuniary gains.

The Greek School of Philosophy, which was led first by Thales and later by other great thinkers such as Hippocrates, Aristotle, and Euclid, made the first attempt the world had known to formulate universal knowledge.

The early Greeks were enthusiastic workers who were exceedingly anxious to explain everything and who had unlimited confidence in their ability to do so, no doubt because there was no one to contradict them by practical demonstration or to question their conclusions.

There were two general methods whereby they could attack their problems. The first was to observe and examine the physical facts available and then to formulate their conclusions on the basis of these facts. The second was to start from some assumption and, by process of speculation, to develop a theory or principle. The Greeks, with their abhorrence of physical labor, naturally chose the speculative method of investigation; and the result was that the more they speculated the more weird their conclusions and theories became. They did not realize that the only way to obtain the secrets of nature is by long periods of observation and experimentation. These men naturally developed theories about which they could not agree among themselves. In trying to establish the origin of everything, Anaximenes said that it was the air, Heraclitus held that it was fire, and Thales maintained that water or moisture was the thing from which everything else came. Thales is supposed to have reached this conclusion because of the importance of water to all animal and vegetable life.

The attitude of the Greeks toward the transmission of knowledge to others is also novel. It is illustrated by Aristotle's reply when he was told, after he had published his lectures, that everyone would then be able to read them and be as wise as his pupils. He said,[1] "My lectures are published and not published; they will be intelligible to those who heard them and none besides." Thus, in his opinion, knowledge was to be for the chosen few and only for them.

The strange thing about the Greek Philosophy is that, even though it was based on incorrect assumptions and presented many strange and unsound explanations of what are now considered every-day occurrences such as the floating of objects and the free fall of bodies, their theories were accepted as true for almost 2000 years in some instances, or until Galileo and Stevinus dared to disprove their claims.

[1] *History of Inductive Sciences*, by Whewell, p. 66.

The characteristic weakness of the Greeks was in great part due to their method of procedure. They expanded a very few facts into wide generalizations, which were as often as not incorrect because of their complete dependence on speculative reasoning.

Aristotle Studies Motion.—The Greek explanation of motion and why it occurred was largely descriptive, rather than factual and specific. Such terms as violent, passive, and natural appear frequently in their literature. The following quotation[2] from Aristotle, explaining why a stone when thrown by hand finally comes to rest, is interesting: "That there is motion communicated to the air, the successive parts of which urge the stone forwards, and that each part of this medium continues to act for some while after it has been acted on and the motion ceases when it comes to a particle which cannot act after it has ceased to be acted upon." This will be observed to be a sort of "lifting by the boot straps" reasoning. The body stops itself rather than being stopped by the resistance of the air to its passage. And Aristotle's incorrect theory that the velocities of falling bodies are directly proportional to their weights is partially stated in the following quotation[2]: "The body is heavier than another which in equal bulk moves downward quicker."

Aristotle attempted to explain the action of the lever in the following manner: He asked, "Why do small forces move greater weight by means of a lever when they have to move the lever added to the weight?" He answered[3]: "Because a greater radius moves faster." Further quotations[3] illustrate his errors in reasoning: "Why does a small wedge split great weights? Is it because the wedge is composed of two opposite levers?" and "Why can a man throw a stone farther with a sling than with his hand? Is it that when he

[2] *History of Inductive Sciences*, by Whewell, p. 71.
[3] Ibid, p. 81, p. 91.

throws it with his hand, he moves the stone from rest but when he uses the sling he throws it already in motion?"

An exceedingly important Aristotelian fallacy was the theory that a continually acting cause or force is needed to maintain motion. He also had no clear conception of density, or specific gravity. False premises no doubt brought about his erroneous conclusions pertaining to falling bodies which, however, kept their authority for about 2000 years.

That Aristotle possessed the ability to recognize and formulate mechanical problems is plainly evident from the foregoing quotations, some of which come from a tract on *Mechanical Problems,* translated into the German by Poselger at Hanover in 1881. Authorship of this tract has been disputed, and it is possible that it should be credited to some other Greek. George Sarton, in his *Introduction to the History of Science,* says of the large number of Aristotelian writings: "Some are genuine but many are evidently the work of his pupils."

Aristotle was the developer of formal logic and he applied his discovery to the theory of science; but, unfortunately, he often took the few available facts and proceeded to the wildest generalizations. Naturally he failed because there were not enough scientifically proven facts available to establish a satisfactory basis for his theories. He did discover the law of the lever for vertical forces, and he proposed some incorrect theories on moving bodies. He is also credited with having written in over twenty-five different fields of knowledge, and this fact in itself would indicate that his investigations in any one field might be expected to be rather superficial.

Prof. Whewell, in his *History of Inductive Sciences,* says: "There is no fundamental truth in the knowledge of science today which can be credited to Aristotle and his followers." Also there seems to be no physical doctrine today of which any anticipation can be credited to Aristotle. Yet Aristotle

is still considered by many to have been the world's greatest collector and systematizer of knowledge.

His importance in the history of science is due to the fact that until the Renaissance no comparable systematic survey of knowledge was made. Aristotle's works are an encyclopedia of the learning of the ancient world, and in every field except physics and astronomy he probably did contribute something of value. He was probably the first to conceive the idea of organized research.

From the standpoint of the physical sciences, if we exclude the work done by Archimedes at a later date, the net result of the labors of the early Greek philosophers was actually less than zero. Not only did they produce no fundamentally sound principles which are now accepted, but they gave—for centuries—their authority to many false premises. Nor does there seem to be any evidence that Aristotle or his followers had any anticipation of what was to come. This last statement is not, however, free from opposition. For instance, Dutens, in his *Origin of the Discoveries Attributed to the Moderns*, attempts to point out that many of the now accepted doctrines can be traced to the work of the ancients. Although there is considerable evidence that scientifically the world would have advanced much more rapidly if there had been no School of Greek Philosophy, such a conclusion is pure speculation and is, of course, open to argument.

If some of the Early Greeks such as Aristotle had devoted their efforts to the study of bodies at rest instead of speculating about the behavior of objects in motion, it is more than likely that they would have arrived at some worth-while results. The world of science might then have benefited materially from their speculations, instead of being left with a lot of vague statements about "natural" and "unnatural" motion which not only were worthless but actually served to confuse and retard the successful development of the fundamental truths of Mechanics.

Should Aristotle then be credited with being the Father of Mechanics, even though his thoughts and speculations on the subject have long since been shown to be incorrect? Should we assume a liberal attitude and say that, since he was the first to offer theories of Mechanics which were accepted by the people of his time as true, he should be credited with being the pioneer in the field? His efforts may at least have served as a sort of catalyst for Archimedes and others whose work was more successful in terms of what the modern Science of Mechanics has learned to accept as experimentally proven facts and fundamental principles.

Archimedes the Thinker.—Archimedes, believed by many to have been the foremost mathematical genius the world has so far produced, not only was a great mathematician but was sufficiently foresighted to put his accomplishments into written form. There seems, however, to be some disagreement in the minds of historians concerning the authenticity of the records now in existence. So well informed an authority as T. L. Heath says, "There now exists no original Archimedian manuscript." He claims that all the information we now have comes from a manuscript which was written in the ninth or tenth century and has since disappeared. Hence, the material now available is probably the result of several reproductions which have been edited more or less extensively.

A large part of the subject matter credited to Archimedes was entirely new to the world of his time. He was an original thinker who did not resort to compiling and systematizing the work of others. Of Archimedes' publications Plutarch says, "It is not possible to find in geometry more difficult and troublesome questions or more simple and lucid explanations." This statement scarcely seems justified by an examination of the work. To modern students the language of the ancients is difficult reading.

It is thought that Archimedes' geometric solutions of problems involving curved surfaces, which were in fact geometrical integrations, furnished Newton and Leibniz with the ideas which led to their development of the calculus.

Archimedes, like Aristotle, put his writings in such form that they are of little value to the uninformed reader. Each step is in its logical order, but explanations of the method of analysis and the reasons why he proceeded as he did are not given. Evidently he was attempting to produce a record for posterity which would serve more as a monument to his ability than as a textbook for future generations to learn from. In fact the manuscripts were of such a nature that it was not until 1880 that a satisfactory translation was made by Heiburg.

It is believed that Heraclides (350 A.D.) wrote a biography of Archimedes' life, but it has long since disappeared. The information now available is rather meager and has been assembled from various sources. He was born in Syracuse in 287 B.C., was the son of an astronomer, and is supposed to have been a relative of King Hieron of Greece. Like the earlier Greek philosophers, he is believed to have spent some time in Alexandria and to have obtained his early training in mathematics in that city from some of the followers of Euclid.

After he returned to Syracuse he devoted his entire life to mathematical research. He is credited with the development of various mechanical devices, such as catapults, cranes, and multiple pulleys, which were later used against the Romans during the siege of Syracuse. He, however, evidently considered these various contraptions as more or less novel playthings which were not important in the life of a mathematician, and so left no written descriptions of them.

Archimedes' life was one of absolute absorption in his studies. He even continued his work through the troublesome period during which the Romans were engaged in capturing the city of Syracuse. There are several stories in existence as

to how he met his death. The generally accepted one is that he was killed by a Roman soldier in 212 B.C. (at the age of seventy-five) because he refused to leave a problem, which he was attempting to solve, and go before Marcellus.

Archimedes Discovers the Principle of Buoyancy. Another story which serves to illustrate Archimedes' complete concentration on the problem at hand is the famous bath-tub incident.[4] King Hieron had commissioned Archimedes to determine whether or not a jeweler had returned a certain amount of gold which had been furnished him for the manufacture of a crown. While taking a bath Archimedes observed that, when the tub was full and he stepped in, his body caused an equal volume of water to overflow. He therefore reasoned that the jeweler might have, in a similar manner, substituted silver for an equal volume of gold. Having hit upon this possible explanation of the jeweler's deception, Archimedes jumped from the tub and ran naked through the streets announcing his discovery.

Archimedes had actually uncovered the jeweler's guilt, but in so doing he had discovered something infinitely more important, namely, the principle of buoyancy. From this experience he reasoned that, when a body is immersed in water, it must raise an equivalent quantity of water somewhat in the manner of weights in a balance. The bath-tub episode seems to indicate that Archimedes had some conception of specific gravity or relative density of materials, but mass and weight were evidently the same to him. To Archimedes, buoyancy was a balance of weights in equilibrium. He so discussed it in his book *Floating Bodies*, in which he states the principles of buoyancy in the following general manner:

(*a*) When a heavy body is entirely surrounded by liquid, it is buoyed up or balanced in part by a force equal to the weight of liquid it displaces.

[4] Vitruvius, in *De Architectura*, Lib. IX.

(*b*) When bodies lighter than the fluid are wholly immersed, they displace an amount of fluid greater than their weight; and so, if left free, they adjust themselves so that they float in a position which will displace sufficient fluid to balance themselves.

(*c*) When a submerged body displaces sufficient liquid to just balance its weight, it will stay at any level at which it is placed.

Another incident also serves to illustrate the personality of the man. He was somewhat of a showman and liked the dramatic. One day he told King Hieron that he could move the earth if he had a place to stand on. The king thereupon asked for a demonstration of his power. Archimedes met this challenge by setting up a compound pulley system with which he moved a loaded ship single-handed. The king was so impressed with the demonstration that he issued an order saying that, from that day henceforth, Archimedes was to be believed in everything he said.

Archimedes' Writings.—That Archimedes must have been a man with great energy, as well as great mathematical ability, is demonstrated by the large volume of original work with which he is credited. The following books are supposed to be the results of his labors:

(1) Sphere and Cylinder.
Two books of sixty propositions demonstrated by geometric proofs.

(2) The Measure of a Circle.
A discussion dealing with the circumference and area of a circle.

(3) Spheroids and Conoids.
A large number of demonstrations dealing with areas and solids.

(4) Spirals.
Twenty-eight propositions dealing with the Archimedian spiral.

(5) Parabola.
Twenty-four propositions dealing with the process of summation; an elementary integration.
(6) Sand Reckoner.
A book on arithmetical numeration.
(7) Collection of fifteen propositions in plane geometry.
(8) Two volumes on the theory of statics and center of gravity (Equiponderants and Centers of Gravity).
(9) Two volumes on the theory of buoyancy and equilibrium of floating bodies.

There is also evidence that numerous other works on various subjects, such as Principles of Numbers, Polyhedra, Center of Gravity, Balances and Levers, Optics, Astronomical Studies, Archimedes' Screw, Wheels and Axles, were produced but have been lost or destroyed.[5] Such a list of work certainly is sufficient evidence that Archimedes was a man of much ability and great energy.

In his works on Mechanics (Equiponderants and Centers of Gravity) Archimedes presents the following ideas:

(1) Equal weights at unequal distances are not in equilibrium but incline toward the weight which is at the greater distance.
(2) When weights at certain distances are in equilibrium and something is added to one of the weights, they are not in equilibrium but incline toward the weight to which the addition is made.
(3) Similarly if anything is taken away from one of the weights which are in equilibrium, they are not in equilibrium but incline toward the weight from which nothing was taken.
(4) When equal and similar plane figures coincide when superimposed, their centers of gravity coincide.
(5) If magnitudes at certain distances be in equilibrium, other magnitudes equal to them will also be in equilibrium at the same distances.

[5] References to some of the lost books will be found in Hermes, Vol. 42; *Bibliotheca Mathematica*, Vol. 7, p. 321; and Bulletin of the American Mathematical Society, May, 1908.

(6) In figures which are unequal but similar, the centers of gravity will be similarly situated. I mean points such that, if straight lines be drawn from them to the equal angles, they make equal angles with the corresponding sides.

(7) In any figure whose perimeter is concave in one and the same direction, the center of gravity must be within the figure.

Archimedes and the Lever.—Archimedes was the first to develop a logical proof for the principle of the lever but, unfortunately, all his experiments were made with a straight lever in a horizontal position. His statements apply only to this special case and cannot be considered as proof of the general principle of the lever. In his discussion of the lever he gives the following axioms and explanations:

(1) Weights which balance at equal distances are equal. For, if they are unequal, take away from the greater the difference between the two; the remainders will then balance, which is absurd. Therefore, the weights cannot be unequal.

(2) Unequal weights at equal distances will not balance but incline toward the greater weight. For, take away from the greater the difference between the two; the equal remainders will therefore balance, and if we add the difference again the weights will not balance but incline toward the greater.

He also demonstrates by similar logic that unequal weights will balance at unequal distances, the greater weight being at the lesser distance.

Archimedes finally concluded that bodies attached to a lever are in equilibrium when their distances from the point of support of the lever are inversely proportional to the weights. As has been previously pointed out, these conclusions were all drawn from investigations made with straight horizontal levers, that is, the forces were always perpendicular to the lever arm.

From these discussions of the action of the lever he proceeds to the problem of the center of gravity. He shows that for two equal weights the center of gravity is at the mid-point of the line joining the centers of gravity of the individual weights. He then proceeds to show that, if three equal magnitudes have their centers of gravity in a straight line at equal distances apart, the center of gravity of the three weights or of the entire system will be identical with the center of gravity of the middle weight.

He defines center of gravity in the following manner: "In every heavy body there is a definite point called the center of gravity in which point we may suppose the weight of the body collected."

The experimental proof of this idea is something which everyone has experienced. The weight of a sack of stones, potatoes, or similar objects is not changed by any rearrangement or change of position of the objects in the sack. Likewise, the weight of any solid object is not changed by changing its position, provided its distance from the center of the earth remains the same.

When these statements are examined, we see that force or pressure is the concept which is essential to the effective study of material objects. Pressure is the measurable effect of bodies at rest, and pressure, weight, etc. are the names used to indicate the vertical downward effect of material objects. Pressure is an effect which is entirely distinguishable from the other effects, such as motion, acceleration, and change of shape. Pressure is present under static conditions, as it also is present during motion.

The Contributions of Archimedes to Mechanics.—Archimedes was unfamiliar with our present conception of the moment of a force; neither did he know anything about the principle of work. Yet he was actually the first, although his contributions to the Science of Mechanics were of a limited

nature, to have any sort of logical understanding of its now accepted basic principles. For this reason many writers consider Archimedes, and not Aristotle, the real founder of the Science of Mechanics. Certainly he was the first to attain even a partially correct knowledge of the laws and facts governing the following fundamental concepts of Mechanics:

(1) Lever
(2) Hydrostatic pressure
(3) Center of gravity
(4) Density

In his opening address before the Fifth International Congress for Applied Mechanics, held at Cambridge, Mass., in 1938, Dr. Karl T. Compton credits Archimedes with the discovery of the principles of statics and hydrostatics, Galileo with the discovery of the laws of motion, Newton with the discovery of the basic principles of dynamics, and Lagrange with the formulation of an equation which stated the laws of Mechanics in a generalized but usable form.

Probably the real reason why no greater progress was made in the field of Mechanics by the Greeks or their conquerors and successors, the Romans—the two dominant powers in Southern Europe—was the abundance of slave labor. There was no real incentive for the development of labor-saving mechanisms or improvement of the standards of living of the lower classes.

The Romans were primarily a military nation and were interested in conquest rather than in the development of the natural sciences. Their contributions to the growth of civilization consisted chiefly of public works, such as baths, large buildings, and roads, which they built without consideration of the factors of economical design since the materials and labor were unlimited; and also of certain improvements in the methods of government, law, language, and the arts. The Roman culture produced no scientific advancement.

Thus, looking back over the period from 400 B.C. to about 500 A.D., we find that very little progress was made in the Science of Mechanics. In fact, if we except the work of Archimedes, the scientific world would have been better off without the early Greek philosophers. Their ideas and the methods by which they arrived at their conclusions led to many misconceptions. This condition, combined with the almost fanatical zeal with which the people were taught to believe the theories presented by the early philosophers, retarded the scientific progress of the world many hundreds of years. Actually no further progress was made in the development of the theory of Mechanics until the time of Galileo and Stevinus, almost 2000 years after the early Greeks began their scientific speculations. The prestige of Aristotelian philosophers during all that intervening period was so great that the ideas which Aristotle and Archimedes had presented lay unquestioned and undeveloped until Galileo and Stevinus had the courage and fortitude centuries later to challenge them.

CHAPTER 3

Medieval Period: 500-1500

Religion and Science.—When the Romans conquered the Greeks and became the dominant power in southern Europe, the Greek culture soon faded. Then followed the heyday of the Roman Empire, which continued for several hundred years. Soon after the fall of Syracuse came the beginning of Christianity in the world.

Dr. Andrew D. White[1] of Cornell University is responsible for the rather dictatorial statement that the establishment of Christianity retarded the development of physical sciences at least 1500 years. There was a widespread belief during the early centuries of the Christian era that the end of the world was close at hand. This was no doubt due to New Testament predictions that the day of the Lord cometh. Many went so far as to predict the exact date of the catastrophe. The general idea was that the year 1000 A.D. would see the finish of all worldly things.

Such theories naturally caused people to abandon science for religion and theology. A shift to religion by large numbers of untrained minds produced many weird theories and beliefs, and gave some not too scrupulous clerics just the material they desired with which to build up what they thought would be an unshakable faith in the powers of religion.

An illustration of these practices occurred during the construction of Monte Cassino (530). Some men were unable to move a stone. They called on St. Benedict who informed them that the devil was holding the stone down. The Saint then proceeded to use his religious powers and was able to lift quite easily by himself a stone which previously six men

[1] *Warfare of Science and Theology*, Vol. I, p. 375.

had not been able to move. Many such happenings were widely publicized during this period. So great was the power of the Church that it became exceedingly dangerous to attempt to explain such incidents as these except in a manner which would enhance the standing of the Church. Any sort of scientific approach was considered blasphemous and brought down punishment upon the presumptuous individual who dared to apply other than theological explanations to such happenings.

Mathematicians were considered heretics and servants of the devil. It was also generally agreed that the earth "standeth fast," for, if it did not, the inhabitants and all the buildings obviously would be thrown off by its motion. Such dogmatic beliefs could lead to but one result: a complete halt of all scientific development. Most of the people devoted their mental energy to trying to prepare themselves for that which was to come after the world cataclysm.

It is not the purpose of this writer to contend with Dr. White and the many others who apparently accept his theory about the effect of Christianity on scientific growth during the early centuries of the Christian era; nor is it his purpose to defend the Church in its attempt to build up its prestige as the all-wise authority on all knowledge—both religious and scientific—in the early centuries of its existence, or to defend it against a more recent philosophy that the acceptance of the teachings of Christianity is incompatible with the highest quality of original thinking, unrestricted study and experimentation in the physical sciences. We shall attempt rather to present the situation as the available material seems to indicate it to have been, and permit the reader to form his own conclusions on these controversial issues. The purpose of this book is to tell the story of Mechanics.

There is little doubt that some of the early scientific workers—Roger Bacon, Galileo, and others—were subjected

to restriction or persecution; but it is difficult to determine whether or not these restrictions were in a large measure due to the clashing of personalities or of minor groups within the Church.

The history of all the more important scientific discoveries or the presentation of new political theories shows that such events have generally been accompanied by much personal criticism, jealousy, and professional opposition by affected groups. To offer so broad a statement as that by Dr. White is dangerous, because there is ample evidence of great scientific accomplishment by many men—laymen and members of the various religious orders—who continued to maintain a more or less harmonious relationship with the Church during part or all of their scientific lives. This was true even of Roger Bacon during many of his productive years, of Copernicus, of Pope John XXII and, in more recent times, of Abbot Mendel, the Augustinian monk who discovered the laws of heredity.

The Galileo controversy with the Church seems to be the classical example that things scientific, especially astronomy, were subjects in which one should not take an active interest—at least not with the intention of changing the theories which the early Greeks had long ago established.

As time went on, however, and world disintegration did not take place as predicted, religious zeal abated somewhat and people began again to try to explain things by logic rather than by religious theory.

Slowly the meager scientific knowledge of the Greeks spread throughout Europe. Much of the work of Aristotle, Archimedes, and their followers had been lost or destroyed during the passage of the centuries, but there still remained enough to furnish the seed for the creation of new centers of learning. About 500 A.D. Constantinople began to be known as a city of great learning. The decadence of the Roman Empire saw a new force developing. During the sixth cen-

tury, the Benedictine monks withdrew from the cities. St. Benedict, in addition to being a religious leader, was a practical man who believed in doing something about conditions which made the religious life of the monks difficult. By going to the country the Benedictine monks obtained the solitude so necessary to their religious well-being. But they were forced by the change to labor in order that they might produce the products required for their bodily comfort. Soon they became skilled artisans who were not ashamed of either their toil or its products. Here we have the first important welding of manual and intellectual labor—a direct reversal of the old Greek belief that to labor was menial. That this new philosophy did not spread rapidly was probably a natural human reaction.

Meanwhile, in the world outside the monastery walls, forces of anarchy and turbulence continued for centuries. It was not until 1000 A.D. that law and order again became dominant. During all these years of turmoil the Latin Church considered the world a hopeless place, from which eternal salvation could be obtained only by prayer and fasting. The reign of Faith had been so absolute during the Middle Ages that no one dared to oppose the voice of religion. During the latter half of the first 1000 years A.D., however, this condition was somewhat modified. About 700 A.D. the Moors went into Spain and started a non-Christian cultural development which was carried on for several centuries.

Moorish Culture in Spain.—Hakim II, Caliph of Cordoba during the tenth century, developed a library at Cordoba of 600,000 manuscripts and maintained buyers in Alexandria, Bagdad, Cairo, and Damascus. The Moors worked in philosophy, mathematics, medicine, astronomy, geography, and mechanics. Their work, however, was not original and consisted of making translations into Arabic from the ancient Greek works of Aristotle and Archimedes and the Egyptian

writings of Euclid. The Moors improved and clarified the principles which had been developed centuries before, but did not produce any original ideas of their own.

The mosques of Cordoba were filled with students. The Giralda Tower at Seville was built as an observatory in 1196, and there were over seventy public libraries in Moorish Spain. Yet Arab learning never seemed to reach the masses; the people were rigid Moslems by whom all men of learning were suspected of heresy. The history of man seems to indicate that an aristocracy of learning has always been more objectionable to the masses than an aristocracy founded on birth and wealth. It has been said that the ignorant are extremely intolerant, but it may be true also that the learned are less ruthless in protecting themselves than aristocrats by birth and wealth. Anyway, the princes of wealth soon learned that an easy way to fortify their position with the masses was to destroy heretical books and manuscripts. This was the fate of most of Hakim's library after his death.

It was comparatively easy for the learned to obtain a hearing in the Moorish mosques where endowed schools usually were maintained. The subjects taught were jurisprudence, logic, philosophy, medicine, mathematics, and astronomy, all of which except jurisprudence the Moors obtained from the Greeks or Arab commentators. The authority of Aristotle among these people may be evaluated from the following quotations taken from Averroes' (B. 1126 at Cordoba) edition of Aristotle. This Arab commentator made the following statements: "Aristotle, the wisest of the Greeks, who both founded and completed the sciences of logic, physics and metaphysics . . . " "No one for fifteen hundred years has been able to add anything to his writings or to find in them an error of any moment." "He should be called divine rather than human."

About 1200 A.D. the Moors introduced Hindu arithmetic and Arabic algebra into Spain.

During the period in which Moorish culture was being developed in Spain, not much of scientific importance was happening in Italy. When the prophesied end of the world in 1000 A.D. had not become a fact, a new, more courageous spirit slowly began to develop. Less time and energy were devoted to religious pursuits, and more to leisure and bodily comforts. A new curiosity about nature and life began to be evident. This change of attitude progressed slowly throughout Europe and England, but eventually during the thirteenth century resulted in the founding of universities at Padua, Bologna, Montpellier, Paris, Salamanca, and Oxford which were outgrowths of the monastic and cathedral schools set up by the Roman Church during the Dark Ages. These schools attracted thousands of students who devoted their energy to the study of subjects of such an abstract nature that little of practical value was developed. It was, according to Lord Bowen (1835–1894), an English jurist, "a good deal like a blind man looking in a dark room for a black hat which did not exist."

The actual formal beginning of the Renaissance can probably be attributed, however, to these early Church schools and to the universities which grew out of them.

It is easier to trace the transition from the old order in the history of philosophy than in the history of science because the history of the period was written by members of the Dominican schools who were not scientists. The thirteenth century, though an exceedingly religious period, appears also to be full of a spirit of free inquiry.

In 1209 the natural philosophy of Aristotle and his commentators was condemned by the Council of Paris, and in 1231 a papal bull by Pope Gregory IX confirmed this action. But, by the middle of the century, Albertus Magnus and St. Thomas Aquinas, two great doctors of the Church, had so rearranged and explained Aristotle's writings that he was again the accepted authority of the Church.

During all these centuries the basic scientific principles which Archimedes had discovered had been safely laid away with their Greek Columbus and his followers. It probably will never be known definitely whether these principles were kept in the background by the great zeal of the religious philosophers or by the fact that Archimedes himself did not know or realize that he had laid the cornerstone of a new branch of science which was destined to be one of man's greatest tools.

However, five years after the Paris Council had rejected Aristotle's philosophy, a boy was born in England who was to advocate a return to the principles of Archimedes; whose name was to be highly honored 700 years later by scientists representing all phases of our twentieth-century scientific development; and who was to become renowned as one of the world's greatest visionaries. This boy was Roger Bacon (1214–1294).

Roger Bacon, the Prophet of Oxford.—In June, 1214, at Dorsetshire, England, Roger Bacon was born. He became a mental genius whose ability to foresee scientific development was far greater than that of any of his contemporaries. The younger son of a noble family, he entered Oxford University in 1226 at the age of twelve. He devoted his entire life to study, living at Oxford for forty years, except for brief intervals at Paris. He was a professional scholar who knew many languages and was considered by many to be the greatest mathematical thinker of his day. However, he functioned most prominently as a prophet of the scientific development of the world, rather than as a doer of specific things.

During his early years at Oxford, Bacon had as his masters Edmund Rich, later Archbishop of Canterbury, and Robert Grosseteste, who became Bishop of Lincoln, and then Chancellor of Oxford and the first Rector of the Franciscan

College. After twenty years of association with the great scholars of the period at Oxford, Bacon—then thirty-five years old—joined the Franciscan order of monks. This step probably was taken because he wished to separate himself from worldly responsibilities and troubles and obtain closer association with the men who had been his mentors, rather than because he had developed greater religious zeal. His becoming a Franciscan friar did not affect his university work. The colleges of Bacon's time were institutions which would gladden the hearts of many of our leading educators of today. Their primary function was to provide a place where the study of the general field of knowledge could be carried on continuously by mature professional scholars. Thus, the college was a production plant of knowledge, rather than a high-powered salesroom for the established trivia of wisdom.

Bacon continued to carry on his studies, and years later (1267) in his *Opus Tertium* he says, "During the twenty years in which I have labored in the study of wisdom I have spent 2000 librae ($3500) on books." When the relative purchasing power of money is taken into consideration, this would mean at the present time about $50,000; therefore, he must have been liberally assisted from outside sources.

There are many references to Bacon and his work in English literature. Like all men who express new ideas, Bacon aroused considerable antagonism. It seems, however, that opposition was not so strong among those of his own generation as in later generations. It was the Dean of St. Patrick's of Dublin, the caustic Jonathan Swift (1667–1745), who is credited with the following comment on the Bacon situation: "When a true genius appears in the world you may know him by this sign, that all the asses are in confederacy against him." Also Oxford, the place of his labors, grew even more critical of his work after he had passed on. The brazen nose of Brasenose College is supposed to be Bacon's nose, and as late as 1818 Lord Byron spoke of Bacon

as having a brazen head. Now that the scientific world has caught up with Friar Bacon, the evaluation of his work has changed. In 1914 Oxford celebrated the 700th anniversary of his birth, and at that time he was internationally honored. This celebration was arranged by the Royal Society in recognition of his ability to foresee the accomplishments of modern physical science.

In 1250 Bacon was forced to retire from public participation in university activities. There seems to be some disagreement among various writers as to the degree of restriction imposed and the reasons for it. According to some sources he was actually imprisoned because of the intolerance of Church dignitaries; others insist that the retirement was merely a restriction of his public activities because of impaired health. Professor Little of the University of Manchester seems to have been able to discover evidence that Bacon was not entirely unproductive during this period of retirement, which ended in 1265. He finds that Bacon did considerable work in optics, in physics, and in the development of his theory for the transmission of force.

In 1265 Guy de Foulques, Archbishop of Narbonne, became Pope Clement IV. A year later (in 1266) Henry III informed the Pope privately that Bacon had made some exceptional contributions to knowledge. Through secret channels the Pope expressed a desire to examine Bacon's accomplishments. At this time his work had not been written up in such form that it could be presented for Papal examination. The request started Bacon to working on his now famous *Opus Majus*, which was followed by *Opus Minus* and *Opus Tertium*. These tasks were completed with considerable difficulty because of lack of assistance, ill health, and opposition from his superiors in the Franciscan order.

In the *Opus Majus* he advanced the theory that light traveled with an appreciable velocity. This theory was opposed to the Aristotelian theory, according to which the

propagation of light was instantaneous, and was probably Bacon's most important scientific discovery. During his labors he had also devoted some time to mirrors, burning glasses, and lenses. Bacon is credited by Lieutenant-Colonel H. W. L. Hine, a British military authority, with the accidental discovery of gunpowder. Hine says that the claims of the Arabs, Chinese, and Hindus will not withstand critical examination, and that the Greek fire was not explosive. These statements are further reinforced by the fact that saltpeter, an important ingredient of gunpowder, was unknown until shortly before the middle of the thirteenth century.

Bacon seems to have been fully aware of the physical importance of his discovery, as well as of the hazardous position in which possession of this knowledge placed him. In order to protect himself from any charges of magic, he recorded the details of his experiments in code.

That Bacon was an apostle of the experimental method of scientific investigation is proven by a chapter in his *Opus Majus*, entitled *Scientia Experimentalis*, in which he says: "Without experiment nothing can be adequately known. An argument proves theoretically but it does not give the certitude necessary to remove all doubt; nor will the mind repose in the clear view of truth unless it finds it by way of experiment." Later, in his *Opus Tertium*, he expresses his conviction in even more forceful language: "The strongest arguments prove nothing so long as the conclusions are not verified by experience. Experimental science is the queen of sciences and the goal of all speculation."

Bacon, however, was not the lone crusader for the scientific method during his life time. Albertus Magnus, the great teacher who rearranged Aristotle's writings so that they again became the accepted scientific philosophy of the Church, was criticized by Bacon with such bitterness that Magnus replied that "some people wrote nothing themselves

but criticized others much." Also, in his book on herbs, plants, and trees Magnus wrote the following: "All that is here set down is the result of our own experience or has been borrowed from authors whom we know to have written what their personal experience has confirmed; for in these matters experience alone can bring certainty—*experimentum solum certificat in talibus.*" A quotation from Magnus' book on minerals continues this thought: "The aim of natural science is not simply to accept the statements of others, but to investigate the causes that were at work in nature for themselves." Bacon is credited with being a severe critic of both Aristotle and Albertus Magnus. Even though Magnus did much to restore Aristotle's prestige, he did not think he was infallible, as is shown by the following statement of his: "Whoever believes that Aristotle was a God must also believe that he never erred; but if one believes that Aristotle was a man then doubtless he was liable to err just as we are." Here we might observe that, if all churchmen had been as liberal-minded as Magnus, there possibly would have been much less ground for criticism of their behavior. But we must recognize that few men, laymen or churchmen, had the mental capacity of Albertus Magnus and Roger Bacon. Therefore, these lesser lights should be given more charitable treatment than has sometimes been their lot.

Bacon, according to Dr. James J. Walsh in *Catholic Churchmen in Science*, attempts to explain the failure of the Latins to make any fundamental progress in knowledge during the Dark Ages by citing their adherence to the following mental attitudes:

(1) Dependence on authority
(2) Yielding to established custom
(3) Allowing weight to popular feeling
(4) Concealment of real ignorance with pretence of knowledge.

Bacon, himself, evidently had little faith in the ability or the capacity of the masses to arrive at any sound scientific conclusions, for Dr. Walsh credits him with the following statements: "Whatever seems true to the many must necessarily be false," and "the common people are not guilty of the fourth fault, concealment of ignorance and assumption of knowledge; that is the peculiar property of the learned professors," and also "authority may compel belief but cannot enlighten the understanding."

As the years pass, we now realize that Bacon was much more of a scientific prophet than an experimental scientist. After his discovery of gunpowder, he suggested that some day man would so control its behavior that he would be able to ride over land and water and fly through the air. This statement suggests his having envisioned the development, in the future, of some sort of successful internal combustion engine. That Bacon also realized that such final results could only be achieved by the successful application of mathematics, the absolutely indispensable tool of the engineer and scientist, is demonstrated by the following statement from his *Opus Majus:* "For he who knows not mathematics cannot know any other sciences; what is more, he cannot discover his own ignorance or find its proper remedies."

In writing of Bacon's appreciation of mathematics, Dr. David Eugene Smith of Columbia University says: "No one in his generation, few men in any generation, certainly no man in mediaeval England, showed such sympathy with mathematics, such familiarity with the standard authors available, such clear perception of the possible applications of the science and such conviction of the value of the subject in a liberal education."

Yet, for many years after his death, Roger Bacon had no prestige whatever. Scholars for generations regarded him as a mere intellectual dreamer without sound factual evidence to support his prophecies. As the years passed and few of his

predictions were realized, Roger Bacon's reputation—or lack of it—remained unchanged. Almost 600 years had to go by before the world even began to catch up with the learned Franciscan monk, and it was only during the last century—more than 700 years after his birth—that his predictions were realized by modern science.

Roger Bacon and Francis Bacon.—It is interesting to examine briefly the world's evaluation of Roger Bacon and of Francis Bacon (1561–1626), Lord Chancellor of England, who lived approximately 300 years later. For some time, Francis Bacon (no relation to Roger), because of his realization that the scholastic philosophy of reasoning had failed to produce any great advancement in man's power over natural forces and also because of his presentation of the experimental and inductive philosophy, had been thought of as one of the greatest intellectual geniuses the world had ever known. Yet, he seems to have had little or no influence on those who were actually carrying on experimental science, with the possible exception of Robert Boyle. Although Francis Bacon, himself, made no outstanding or successful contribution to our knowledge of natural phenomena, nevertheless his writings seem to have impressed some historians profoundly.

Francis Bacon even refused to accept the Copernican Theory, nearly a century after the death of Copernicus. This is evidence of the limitations of his scientific knowledge and ability, but these limitations may have been the result of lack of familiarity with the knowledge possessed by continental Europe. Most informed historians now feel that Francis Bacon was given much more prominence than his accomplishments rated. This is the opinion of Dr. Whewell, the noted English writer on History of Science. The following evaluation of the two Bacons is from the pen of Bridges, the English editor of Roger Bacon's works: "Between the

fiery Franciscan, doubly pledged by science and by religion to a life of poverty, impatient of prejudice, intolerant of dullness, reckless of personal fame or advancement, and the wise man of the world, richly endowed with every literary gift, hampered in his philosophic activity by a throng of dubious ambitions, there is little in common. In wealth of mind, in brilliancy of imagination Francis Bacon was immeasurably superior but Roger Bacon had the sounder estimate and the firmer grasp of that combination of deductive with the inductive method which makes scientific discipline. Finally Francis Bacon was of his time; with Roger Bacon it was far otherwise."

It is now generally admitted that Roger Bacon's freedom was restricted by the Church, but to what degree is rather uncertain. As is often the case with men of his type, Roger Bacon was not wise in controlling his pen or his speech and was not amenable to discipline. He was bitter and personal in his attacks on those—both laymen and his brother Franciscans—whom he knew to be less well informed than himself. At times he was so indiscreet as to loose his arrogance against other religious orders, and such actions resulted in further arousing the animosity of his Franciscan brothers and stimulating his rivals to greater efforts against him.

As long as Bacon's criticisms were directed against members of his own religious order, it was only a family affair; but, when he let his arrows fly in the direction of the Dominican brothers, his behavior became something which his superiors of the Franciscan order could no longer disregard. It then became necessary for the Minister General of the Order of the Franciscans, Jerome of Ascoli, who later became Pope Nicholas IV (the first Franciscan Pope), to issue an order placing Friar Bacon in enforced retirement. The records show that it was on the "advice of many of the Franciscan brethren that the doctrines of the English Brother Roger Bacon were condemned and rejected." It therefore seems

that whatever limitations to his physical and mental activities Roger Bacon was forced to endure resulted entirely from his own indiscretions and were imposed through the members of his own order, even if the action may have been inspired by some undisclosed directive from the higher officials of the Church at large.

Part II

Scientific Thought Begins To Function

CHAPTER 4

The Renaissance: 1400-1600

Why the Greeks Failed.—Except for the elementary beginning made by Archimedes, the world had no natural science before Roger Bacon's time. The very conception of a natural science implies theory compared with and controlled by observation and experiment. The science of Aristotle was purely speculative and dogmatic. Professor Whewell[1] refers to the errors of Aristotle's science as not due to neglect of facts but to "a neglect of the idea appropriate to the facts, the idea of mechanical cause, which is force, and substitution of vague or inapplicable notions, involving only relations of space or emotions of wonder." His was a science established with the authority of a religious creed and founded on erroneous ideas and the relations of words and axioms. As religion claimed full authority over her subjects, so this philosophy claimed imperial and despotic power over her doctrines, and disagreement could neither be blameless nor allowed. To err was wicked, and to reject was heresy and became almost equivalent to a renunciation of Divine power.

But, what stopped the progress of the Greeks after they had broken away from useless speculation by "the internal light of the mind alone" and introduced directed experimentation with instruments and observed facts? Why had it taken centuries to bring about a practical approach to natural phenomena? Professor Whewell in his *History of the Inductive Sciences*, Vol. I, offers four causes:

(1) Obscurity of thought
(2) Servility
(3) Intolerance of disposition
(4) Enthusiasm of temper

[1] *History of the Inductive Sciences*, p. 83.

After the fall of Greece and the decline of the Roman Empire the large cities had fallen into a normal decline. Christianity arrived, and with it came the hardships which the early Christians were forced to endure and which must have produced a state of mind not at all suitable for the impersonal analytical type of thinking required by scientific research.

It appears that scientific progress not only requires, but demands, a certain method of life. If science is to grow, there must be an established, well organized and permanent urban society, a prosperous industry, and a transportation system which is adequate. The conditions which existed during the Middle Ages did not meet these requirements. A civilization which devotes most of its energy to the strengthening of its political and military status through the construction of improved roads, bridges, harbors, and public buildings, and which conducts agricultural pursuits on a subsistence level, is not going to produce much that can be classified as scientific in nature.

Aristotle Re-established by Magnus and Aquinas.—The Renaissance had its birth in the Italian schools of the early decades of the twelfth century. This educational revival continued with the establishment of the Italian universities and the rejection and re-establishment of the Aristotelian philosophy as the scientific creed of the Latin Church, at about the same time at which Roger Bacon was spreading the leaven of revival at Oxford by encouraging the study of the dead languages, philosophy, and mathematics.

Oddly enough, no single incident had a more potent influence in the rehabilitation of sound basic scientific thinking than the re-establishment of Aristotle as the apostle and Lord High Chancellor of scientific thought, even though the new version was considerably different from the original work of Aristotle. The foremost interpreters of

Aristotle were the Dominican, Albertus Magnus of Cologne (1206–1280), who brought together Aristotelian, Arabian, Jewish, and Neo-Platonic thought, including all the contemporary knowledge, and his famous pupil, Saint Thomas Aquinas (1225–1274), who carried on Albertus' work of rationalizing the existing knowledge. It was the authoritative acceptance of Saint Thomas' Aristotelianism by Christianity which mainly was responsible for the hostile attitude of the Roman Church toward the development of experimental science.

Saint Thomas' greatest works were *Summa Theologiae* and *Summa Philosophica contra Gentiles*. These two books were supposed to contain the Christian knowledge for the ignorant drawn from two sources, namely, the Scriptures and the natural wisdom of Aristotle and Plato. Since both sources stemmed from God, theology and philosophy could not be antagonistic. Saint Thomas dealt with all knowledge in terms of God, the Scriptures, and Aristotle. He attempted to do a rational synthesis of all knowledge based on Aristotle's science and logic and the authority of the Catholic Church. His work therefore was entirely unsuited to the stimulation of experimental investigation. He assumed that the natural phenomena of the world should be described in terms of human psychology and sensation, which was the foundation of Aristotle's physics, his weakest scientific subject. This was opposed to the theory, more in agreement with the modern concept of physics, which had previously been presented by Democritus (born 460 B.C.), according to F. A. Lange in his *History of Materialism*. Democritus is credited with the formulation of the theory of atoms and molecules. Roger Bacon thought him more able than Aristotle or Plato. The modern physical viewpoint of Democritus is evidenced by the following quotation[2] and principles ascribed to him: "According to convention there is a sweet

[2] *History of Science*, by Sir William Dampier, p. 95.

and a bitter, a hot and a cold and according to convention there is colour. In truth there are atoms and a void." John Tyndall, in an address "Advancement of Science" (1874) before the British Association for the Advancement of Science, credits Democritus with having deduced the following principles:

(1) "From nothing comes nothing. Nothing that exists can be destroyed. All changes are due to the combination and separation of molecules.

(2) "Nothing happens by chance. Every occurrence has its cause from which it follows by necessity.

(3) "The only existing things are atoms and empty space; all else is mere opinion.

(4) "The atoms are infinite in number and infinitely various in form; they strike together and the lateral motions and whirlings which thus arise are the beginnings of worlds.

(5) "Varieties of all things depend upon varieties of their atoms, in number, size and aggregation.

(6) "The soul consists of free, smooth round atoms like those of fire. These are the most mobile of all. They penetrate the whole body and in their motions the phenomena of life arise."

When we realize that these are the thoughts of a man who was born about 100 years before Aristotle and that within the short span of approximately three centuries Pythagoras, Democritus, Plato, Aristotle, Euclid, and Archimedes left their imprint on the world, it seems incredible that the human race should have had to wait 1500 years before it could produce those who would seriously question or add to the accomplishments of these men.

Beginning of the Revolt Against Scholasticism.—With the completion of Saint Thomas Aquinas' work the prestige of what has long been known as Scholasticism reached its highest point. Even though its position remained strong, it

could not be expected that such contemporaries of Saint Thomas as Roger Bacon would accept his writings without criticism. However, Bacon's criticism was far in advance of the thought then current and consequently produced little but trouble for Bacon himself, since the followers of Saint Thomas Aquinas believed that this great churchman had attained a union of philosophy and religion. This belief did not continue unchallenged long, for with the end of the thirteenth century and the beginning of the fourteenth century Duns Scotus (1265–1308) of Oxford insisted that even a larger part of theology than the part described by Aquinas was beyond demonstration by reason. Scotus maintained that the Christian doctrine was the arbitrary Will of God and that the free will of man was his primary attribute and much more important than his ability to reason. Here was the beginning of the revolt against the domination of philosophy by religion as it had been set up by Saint Thomas Aquinas.

The writings of William of Occam, also an Englishman (who died in 1349), were even more radical than those of Scotus. He insisted that no theological doctrine was demonstrable by reason, and he objected to the papal supremacy and led the Franciscans in a revolt against Pope John XXII. The efforts of Occam were the beginning of the end of Scholasticism. Philosophy was now about to start its march toward the goal which would eventually free it from all domination by the theologians and permit it, by means of its union with experimentation, to develop a natural science.

The Causes of the Scientific Renaissance.—As both the explanations of theology and the wisdom of the ancient Greek philosophers were found wanting, the power and influence of the Church in scientific matters grew less and less. During this period the compass and gunpowder, those two great tools of exploration and conquest, had been

invented; the first book, the Gutenberg Bible (1455) had been printed; and the western hemisphere had become part of the known world. As domination of the Church over intellectual matters receded, there inevitably was developed a desire to learn more about the cause and effect of the many and varied physical and social phenomena which men were continually experiencing in their daily lives. The people were no longer satisfied with the dogmatic explanations of the supposedly all-wise clergy or with the weird and sometimes illogical theories which the Greek philosophers had handed down in their writings.

During the period when the Church was all powerful it was only natural that it should attempt solidly to entrench its financial and physical position. This condition probably explains why so many of the cathedrals dated from this period. While these structures are monuments to the artistic development of the times and the power and wealth of the Church, they exhibit no evidence of scientific or engineering knowledge, except possibly the arch and buttress. Many were designed by men of great artistic ability, but all were built by tradesmen whose knowledge of building was limited to what they or their predecessors had learned by the method of trial and error.

Scientifically, nothing of major importance had been developed since the beginning of the Christian era. For, as Sir William Dampier[3] says, "The Christian Middle Ages are weakest in the special department of thought necessary for scientific enquiry. We have but glanced at their work of forming and consolidating the nations of Europe. We have not touched on their wonderful achievements in literature and art." Lest this be considered too strong an indictment of the Christian religion, we must remember that, while it is true that the birth of Christianity did slow up the development of science, it is also true that today the world owes

[3] *History of Science,* p. 105.

practically all its cultural and scientific development to those regions where Christianity is strongest. At the inception of Christianity many changes—social and political—had to be made before conditions conducive to scientific progress were established.

We of the present generation are in the opposite situation. The past century has brought forth spectacular scientific developments and the years ahead give promise of equally great results. But what meanwhile of our political and social conditions? The curve of scientific development has been going up much more rapidly than that representing our political and social status. At the present time it appears that the world needs another Renaissance. This time not to reinforce or stimulate science but to attract capable vigorous men to the study and development of our political and social problems, so that these may again move forward at approximately the speed at which science and industry are traveling.

Beginning of the Renaissance.—The efforts of Duns Scotus, William of Occam, and Roger Bacon were important influences toward the development of revolt against the control of the Church over things scientific. Other signs of the Renaissance were soon to appear in Italy. In the northern part of the country a prosperous and vigorous people began to build a leisured and intelligent culture in the cities, which was a most desirable environment for the starting of the Renaissance. There soon developed an interest and enthusiasm which brought about a widespread search of the cathedral libraries of Italy, Northern Europe, and the East for hidden Greek manuscripts. Such efforts could have only one result: a spirit of free investigation and study developed, which soon found its way into all fields of human knowledge. However, the authority of religion had dominated men for so long that they found it easy to accept direction in matters outside the religious field. With the weight of religious

authority behind the Greek philosophers, it was difficult for the revival of science to get started.

At one period in the early part of the sixteenth century The Vatican, itself, was a stimulating center for the study of the ancient culture, but after the capture of Rome in 1527 by the Imperial soldiers the Vatican began to be antagonistic to such cultural development, especially when such great progress had been made that it could no longer control or understand the newly developed learning.

By this time Columbus had discovered America, Magellan had circumnavigated the globe, and the gold from the new world was supplying the money needed for the widely expanding commerce. Trade became profitable, and the wealth and leisure necessary for intellectual pursuits increased and became widespread.

Sir William Dampier[4] calls attention to the fact that the three periods in the history of the world noted for their intellectual fecundity—the great Greek period, the Renaissance, and the present time—are all periods during which there was great geographical and economical expansion accompanied by increased wealth and opportunities for a leisurely cultured life. High-speed transportation and communication in our period have taken the place of geographical expansion. In Greece slavery was the factor which produced the wealth and leisure; during the Renaissance it was the wealth of the Indies; while during the present century it has been the mechanization of industry. The Greek culture came to an end because of the political collapse of the country. The Renaissance was the beginning of a period of approximately four centuries during which the power and population of the European countries increased rapidly. This great increase in population naturally provided many more trained minds than the Greeks had at the peak of their cultural development. A similar increase may also be a

[4] *History of Science*, p. 111.

partial explanation of our present great scientific success—simply many more minds at work rather than any vastly greater individual mental power. It may be well for modern science to keep this fact in mind when it boasts a bit about its accomplishments.

It is difficult to look back over the years and pick out the factors which finally brought about the scientific Renaissance, but the following may be mentioned as contributing greatly:

First, there was the greatly increased quantity of gold, and with it a steadily rising scale of prices.

Second, with the development of the art of printing the transfer of knowledge became a much more general and easily accomplished task than when it was limited to word of mouth or the hand-produced manuscript.

Third, that a few prominent and able thinkers should be convinced that the written word of Aristotle and his followers as presented by St. Thomas Aquinas and the Church needed further investigation and examination was not sufficient; it was necessary that the large numbers of trained people should be awakened to the importance and value of the scientific truth before a scientific Renaissance could get under way.

Fourth, the development and application of power generated by other means than the muscles of man or beast; many of the primitive machines, some operated by muscle power and others by water power, may be found illustrated in *Theatorum Instrumentorum et Machinarum,* by J. Benson, published in 1578.

Leonardo da Vinci, a Man with an Open Mind.—Leonardo da Vinci (1452–1519) started in life as the natural son of a lawyer of considerable prominence, Ser Piero da Vinci, and a peasant girl, called Catarina, but he overcame his handicap of birth—if it was a handicap—and, through his

personal charm, power of mind, and character, became the friend of statesmen, princes, and the wise men of his time. Because of his friends and his paintings, he was one of the great personalities of the Renaissance.

Leonardo da Vinci had long been known as a great painter and sculptor, but the world did not realize his mental power and genius as a scientist until almost four centuries after his death in France in 1519, because he never published his scientific conclusions. It was not until 1906 that the first English translation of da Vinci's notebooks was made by Edward McCurdy.

He was an able architect, physicist, biologist, philosopher, and engineer. He was not a Scholastic; neither was he a follower of the classical authority, as were many of his contemporaries. Rather, he believed that the knowledge of the ancients should be used for scientific reference only in so far as it could be proven by experiment and observation. His approach to science was that of the modern scientist. He regarded the teachings of the Greeks and the theologians as good to know, but he also sought to find out for himself how nature behaved. His was an open mind not to be impressed by the doctrines of theology unless they seemed to be logical; he often attacked the abuses of the Church, but was willing to accept the principles of Christian teachings as good. His feeling toward Christianity he expresses in the following words: "I leave on one side the sacred writings because they are the supreme truth." He was fortunate that he lived at a time when the Church of Rome was liberal. Had he come along a few years later when the reactionaries were in power he, no doubt, would have experienced some of the difficulties with the Church which came to his followers.

Thomas Aquinas' synthesis of Scholasticism had made man conscious that the behavior of nature was something which could be understood, but da Vinci soon learned that

the explanations offered by Scholasticism were not entirely satisfactory and that the truth had to be obtained by observation and experiment. It was only natural that the old work of Archimedes should now again become interesting, and in 1543 Tartaglia published a Latin translation of some of Archimedes' manuscripts.

Leonardo had access to some of these and applied his experimental procedure to some of the problems, but he published nothing about his work and left only his personal notes. Da Vinci was not the originator of the experimental procedure. Alberti (1404–1472), a student of mathematics, had made physical experiments.[5] However, it was not until Galileo actually put the experimental method into practice that its value and importance in scientific research were recognized.

Da Vinci appreciated the value of mathematics as a tool for the development of science, but he also said:[6] "Those sciences are vain and full of errors which do not end with one clear experiment." From this statement it is evident that Leonardo da Vinci believed that science, to survive, must start from observed facts or experimentally determined data. The principles of mathematics may then be applied to check and prove the observed facts and to justify the conclusions, but experimentation should be the basic approach if the investigator is not to be led astray as were the Greeks of Aristotle's time.

Leonardo da Vinci seems to have been the first to grasp the idea which has since become known as the statical moment of a force. Both Aristotle and Archimedes confined their attention to parallel vertical forces acting on a horizontal lever, but da Vinci realized that there was more to be learned about the lever than was involved in that simple case.

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[5] *History of Science*, by Dampier, p. 114.

[6] *Ibid*, p. 115.

As early as 1499 he had determined correctly the magnitude of the oblique force required to support the weight W on the lever in Fig. 1.[7] Having solved this problem, he then proceeded to construct an apparatus[8] similar to that shown in Fig. 2. Thus, a full century before Francis Bacon began his crusade for the experimental method, da Vinci had used

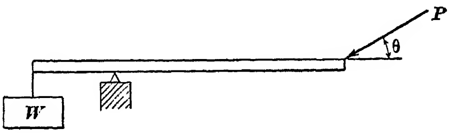

FIG. 1

it effectively. Da Vinci suggested the following problem for Fig. 2: "What should be the ratio of the weights W_1 and W_2 if equilibrium of the lever AB is to be maintained?" He proceeds to explain the problem in the following manner.

The lever arm of the weight W_1 is not AB but is the potential lever arm AC. The lever arm of weight W_2 is also not AB but is the potential lever arm AD. Just how and in what manner da Vinci arrived at these deductions cannot be determined, but it is clearly evident that he recognized the effective (or, as he calls them, the potential) lever arms of the weights.

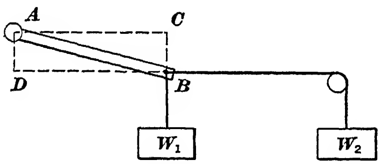

FIG. 2

Some writers, notably P. Duhem (1905) and E. Wohlwill (1909), have attempted to show from their studies of old books and manuscripts that the scientific progress made during the Renaissance was simply the slow and gradual develop-

[7] *History of Inductive Science*, by Whewell.

[8] *Science of Mechanics*, by Mach.

ment of the ideas of the ancient Greek philosophers, Aristotle and Archimedes. They also seem to be of the opinion that, while da Vinci's manuscripts were not published until a long time after they were written (and then only in part by Venturi in 1797), it is entirely possible that in the intervening period many people had access to them and that the information contained therein thus reached and influenced other investigators. If da Vinci had published his work in a form that would have made it available to all, the Science of Mechanics might have been advanced by 100 years; it would not have had to wait for Galileo.

Some of da Vinci's manuscripts are now in the possession of the Royal Library of Paris. Examination of these shows that, in addition to understanding the problem of the lever, da Vinci was also familiar with the concept of work. He does not call it work, but says that "when a force raises a body through a definite path in a certain time, the same force can raise half the body or weight in the same time through a path double in length." He applies this theorem to levers, pulleys, and machines in such a manner that it is easy to determine exactly the idea he wished to convey. Another example given is: "If we have a definite quantity of water we can cause it to operate one mill of a definite size or two mills of the same size as the first but the two mills will accomplish only as much as the first mill." He seemed also to have had some instinctive realization of inertia, for he says: "Nothing perceptible by the senses is able to move itself; every body has a weight in the direction of movement." He also knew that the speed of a falling body increased with the time, and that the ratio of the time of travel down an inclined plane to the time of vertical fall through the same height was the same as the ratio of the distances involved.

Many of Leonardo da Vinci's writings contain errors. It must be remembered, however, that these manuscripts did not represent a formal presentation of the various subjects

but were largely private notes and sketches made at different times—probably notes made when the ideas first came to him. Certainly they are not thoroughly digested and formulated principles made for formal presentation.

The germs of other ideas, such as the impossibility of perpetual motion, may also be found in the manuscripts. He says: "No body can, by its motion of falling, rise to the height from which it fell; its motion reaches an end." He also says: "Force is a spiritual and invisible power which is impregnated in bodies by motion; the greater it is, the more quickly does it expend itself."

It is quite possible that these manuscripts may have furnished the seed which grew into ideas that influenced the work of other men. If this is the case, the later investigators did not give Leonardo da Vinci credit for inspiration or assistance.

As Dr. Ernst Mach says, "Natural science grows in two ways: first by our retaining the observed facts and processes; and then by our reproducing or reconstructing them. During this process of reconstruction errors and additional information will be observed. The second step in the development of natural science is then the correction of previous errors and the logical consolidation of observed facts."

Consideration of the process of growth makes it at once evident that every scrap of proven and digested knowledge which can be passed along from one generation to the next makes the speed of travel along the road of scientific growth that much more rapid. However, if the transmitted information is incorrect, fallacious theories may result, as in the case of the Greeks.

Copernicus and His New Planetary Theory.—Nicolaus Koppernigk (1473–1543), or Copernicus, as he was later called, was born at Thorn, Poland. His father was supposed to have been a German.

He studied the arts, astronomy, and medicine at Cracow and then went to Bologna for his mathematical training. Soon after finishing his work at Bologna he was appointed professor of mathematics at Rome. There he became interested in the affairs of the Church, took the "orders," and became a monk. From Rome he returned to Thorn, his home, where he was priest of the principal church and later became a Canon of the Church.

In addition to carrying on his religious work, Copernicus was a serious and able student. He was especially interested in astronomy. Up until this time the only accepted theory of the planetary system was that known as the Ptolemaic. According to this system the earth was the fixed center of the heavens and all the other planets revolved about the earth. Numerous strange devices were advanced to explain the operation of the Ptolemaic system. One scheme had the various bodies set in concentric crystal globes somewhat as jewels. These globes were then supposed to rotate with the earth as the center.

After many years of study Copernicus evolved an entirely new planetary theory. According to his theory the sun was the center of the entire system and all the other bodies rotated around the sun. Copernicus realized that his ideas were so revolutionary that they would cause repercussions; so he did not publish them. Thirty years later a Cardinal of the Church became interested in the manuscript. He paid the publication cost and had the ponderous volume dedicated to the Pope. The book was so large and the subject matter so difficult to understand that few undertook the task of reading the volume. Thus, the new theory did not become widely known for a considerable period of time after its publication, even among those who were scientifically inclined.

The dedication of the volume to the Pope placed the Church in a rather difficult position. When in the course of

time the exact content of the book was learned, the ruling powers of the Church immediately realized that, if this new theory became generally known, the entire existing conception of the behavior of the planets would have to be changed. The seven days of the week had been named after the seven planets. The Middle Ages had inherited this by virtue of the stubborn vitality of astrology, itself, but the sanctity and incorruptibility of the astrological numbers was made unquestionable by their presence on page after page of Holy Writ which had been pondered over and expounded by generations of churchmen. The dignitaries of the Church therefore declared the book forbidden reading, and those who believed in the new theory were considered heretics and were punished by torture and in some cases by death.

While Copernicus, himself, made no important contributions to the Science of Mechanics, the controversies that arose over his planetary theory did much to awaken a new interest in science in general and no doubt were an exceedingly important contributing factor in the development of the new era which followed soon after the publication of his planetary theory. Such often are the repercussions of events which, at the time they occur, seem to be entirely dissociated from, and to have no influence on, other events; when later we examine the picture more closely, we are able to see the connecting link.

CHAPTER 5

The Beginning of the Modern Period

Further Progress in the Fundamentals of Mechanics. The reawakening of scientific thought brought about during the Renaissance Period (1400–1600) naturally carried over into the period of the actual development of science. We have seen how the work of Copernicus, probably helped some by that of Leonardo da Vinci, served to awaken new interest in the Science of Mechanics. It was, however, the publishing of the new planetary system by Copernicus which was the actual catalytic that started this new era of independent thinking. The publication of his book gave courage to many thinkers to go ahead with their work independently, disregarding the edicts and theories of the Church and Aristotle, and to try to work out and explain the basic principles of science by logic and experimentation.

While actually having little or no immediate connection with Mechanics, the publication by Copernicus seemed to be just the spark needed to kindle the fire of enthusiasm for the further quest of the truth. It gave scientific development a tremendous impulse which is clearly demonstrated by the number of important names that appear in the historical records of the period just following the publication of the book. Copernicus was indeed the knight in shining armor who destroyed the myth of the infallibility of Aristotle and started the revolution against the dictatorship of the Church over things scientific. The scientific world of today owes much to him. The battle was not won in a day, but continued almost to the year 1700 before the foes of the new order finally succumbed.

Writings by Stevinus.—The next important contributor to the development of the Science of Mechanics was Simon

Stevinus (1548–1620) from the town of Bruges, a military engineer with considerable ability as an experimentalist. He published two books: *Beghinseln der Weegkonst* at Leyden in 1586 and *Wisconstige Gedachtenissen* at Leyden in 1605. A Latin translation called *Hypomnemata Mathematica* by Willebrord Snell was published in 1608; and a revised edition in French by Albert Gerard was published in 1634. In *Hypomnemata Mathematica* under Statics, Stevinus discusses the following subjects:

(1) Elements of statics
(2) Practical statics
(3) Theory of the center of gravity
(4) First principles of hydrostatics
(5) Practical hydrostatics
(6) Miscellaneous topics

In *Hypomnemata Mathematica*, Stevinus attempts also to define what are now considered some of the fundamental terms or concepts of Mechanics. Statics is defined as the interpretation of the computations, proportions, and conditions of equilibrium and of weight. Weight of a body is defined as its force of descent in a given place.

Stevinus was also one of the many who attempted to solve the problem of the lever. He was more fortunate than his predecessors in that he applied his efforts to the general case, that is, the lever with forces acting in any direction. However, his most important contribution to the Science of Mechanics was his demonstration of the principle of the inclined plane through the composition and resolution of forces by the parallelogram of forces. His writings contain many problems for which he obtained a solution by employing this method, but nowhere does he offer a proof of the principle of the inclined plane.

The Inclined Plane.—Stevinus' analysis of the inclined plane was simply an exposition of how it worked, not a

mathematical proof. He formulated the following principle from his observations:

If a plane triangle is placed vertically with the base parallel to the horizon, and upon the other two sides are placed single spheres which are in equilibrium, as in Fig. 3,

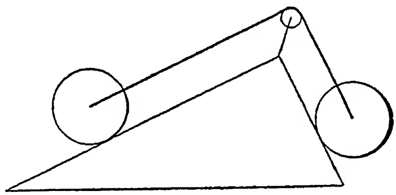

FIG. 3

then, according as the right side of the triangle is to the left side, so is the balancing effect of the left sphere to the counter-balancing effect of the right sphere.

These conclusions were arrived at through the following demonstration: First he considered a block, such as *A* in Fig. 4, on a horizontal plane. For such a block the weight is directly opposed by the upward reaction of the plane on the block. Here a condition of equilibrium exists and the block

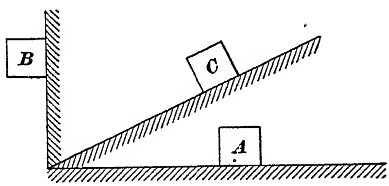

FIG. 4

remains at rest. For the block *B* on a smooth vertical plane an entirely different condition exists. Here the weight of the block is unopposed by any force from the smooth plane. Therefore, the weight moves downward under the action of gravity on the unopposed weight. Next Stevinus investigated the block *C*. That he might study this case he constructed an endless cord or chain with 14 equally spaced balls

of equal weight attached to the cord, as shown in Fig. 5. Any cord or chain of uniform weight per unit length would have served the purpose just as well and would have been less troublesome to work with.

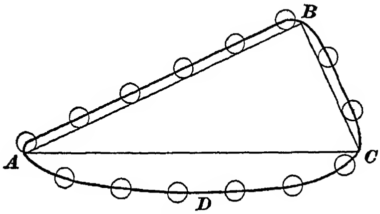

FIG. 5

Stevinus assumed that the chain in Fig. 5 or the cord in Fig. 6(*a*) when placed over the triangular prism would be either in equilibrium (at rest or stationary) or not in equilibrium (moving with an acceleration due to unbalanced force). If it is not in equilibrium the chain or cord would move when placed on the prism. Since the movement of the chain or cord would not change the system of forces which act on the chain or cord, the chain or cord would then continue in

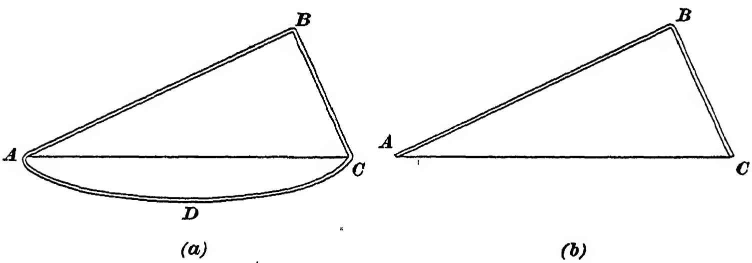

FIG. 6

motion. Such movement would result in a state of perpetual motion, which Stevinus knew was impossible. He therefore decided that equilibrium must exist when the chain or cord is placed on the prism. Since the part ADC of the chain in Fig. 5 or the cord in Fig. 6(*a*) is clearly in equilibrium, it follows that the lengths AB and BC must also be in equilibrium or that they balance each other.

The reader or student will generally be satisfied with this demonstration. But why? If he examines the solution carefully, he will realize that the answer has been obtained because the original assumption (equilibrium of the complete chain) was known instinctively to be correct. Stevinus presents no mathematical proof of this basic assumption. He simply accepts it, just as the reader does.

This method of developing fundamental principles of science from instinctive knowledge is part of the process of evolution of many of our modern sciences. We must possess or form some basic conceptions upon which to build up our scientific structure. The better our foundation (the instinctive knowledge or assumptions), the more certain we are of building a superstructure which will stand the assaults of time and the logical developments which follow as the science grows.

Further examination of this problem presented by Stevinus discloses that it demonstrates also the principle of work; that is, for every given weight which descends a given vertical distance, an equal weight ascends a like vertical distance. Stevinus in his experiments with pulleys observed that the same relationships existed. He did not, however, see any connection between these two demonstrations; or, if he did realize that the conditions were similar, he made no formal statement of any basic principle involved in the two cases.

Stevinus was exceedingly fortunate in his selection of the endless chain or cord as a mechanism with which to work. Such a chain, we know instinctively, is always at rest or in equilibrium in whatever manner, shape, or form the loop is supported. This then was a sound base from which to start. From this general case Stevinus was able to develop the rules or conditions which must be satisfied for equilibrium of weights when the weights are placed on specific planes.

Archimedes was not so fortunate when he did his work on levers, since he confined his efforts to the special case of parallel vertical forces and therefore was unable to formulate the rules for the general case of equilibrium of any lever.

By solving the problem of the inclined plane, Stevinus not only had obtained the answer to a problem not previously solved but had developed a new technique for such solutions which has been exceedingly useful in the development of science in general. This was the idea of starting from an instinctively admitted or acknowledged fact as a base and

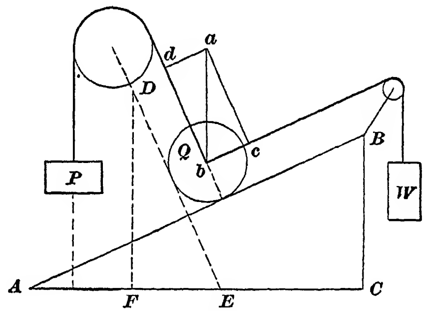

FIG. 7

then applying logical reasoning to the further development and solution of an otherwise insoluble problem. Stevinus evidently was fully aware that this was an important advance in scientific procedure because on the title page of his book *Wisconstige Gedachtenissen*, published at Leyden in 1605, he has placed a decorative vignette showing the prism supporting the chain with the fourteen balls. The picture carries the following inscription: "*Wonder en is Gheen Wonder.*"

The Parallelogram of Forces.—After determining the principle of the inclined plane, Stevinus began to apply this principle to the solution of other problems. He constructed what has since been called a funicular machine and proceeded to demonstrate the law of the composition of forces.

In Fig. 7 the weight Q is held in equilibrium on the plane AB by the weight W. The weight P is then attached to Q so that the direction of the cord Qd is normal to the plane AB; and P is adjusted so that it is just great enough to support Q. The plane AB could then be removed if desired. The system of cords and weights is in equilibrium, and the form of the inclined plane is still retained. Next the vertical line ab is drawn so that its length represents the weight Q to scale. Then from point a the perpendiculars ac and ad are drawn as shown. Since the form of the inclined plane is still maintained, $\frac{W}{Q}=\frac{BC}{AB}$; and, from the similar triangles,

$$\frac{W}{Q}=\frac{BC}{AB}=\frac{bc}{ab}$$

This demonstrates that W, which represents the component of the weight Q parallel to the plane AB, is represented by bc or ad to the scale to which ab represents the weight Q. In a similar manner, if Q is supported on the inclined plane DE, then

$$\frac{P}{Q}=\frac{DF}{DE}=\frac{db}{ab}$$

Here again, unfortunately, a fundamental principle has been demonstrated by showing its application to a special case rather than by making the application to a general case. Stevinus has demonstrated the parallelogram of forces by using the special case where the component forces are perpendicular to each other. In some of his later work he employed the more general form of the parallelogram construction (where the components are not perpendicular to each other), but how he demonstrated the validity of this more general construction, if he did, must remain unknown. He left no proof or explanation. He does, however, use the construction correctly for determining the tension in each of three cords which meet in a point as in Fig. 8.

In this demonstration of the parallelogram construction for resultants and components, Stevinus has given us no exact mathematical proof of the validity of the construction. He simply demonstrates that the construction does work, just as he did with the inclined plane. Furthermore, his actual demonstration was performed by application to the special case of components at right angles. He then proceeded to use the construction for problems where the components were not at right angles.

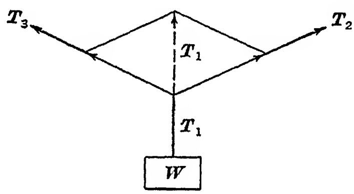

FIG. 8

Here again we are presented with the case of something being accepted as true and valid when no proof has actually been given. The first correct general statement of the principle of the parallelogram of forces did not appear until long after Stevinus had died. This statement was finally given by Newton in his *Philosophiæ Naturalis Principia Mathematica* published in 1687 in London, and also in the same year by Varignon independently in a paper sent to the Paris Academy but not published until after the author's death, and again in the same year (1687) by Father Bernard Lami in an appendix to his *Traité de Mécanique.*

Newton arrived at his conclusions through a study of the motion of a body caused by two independent forces acting during the same interval of time. He found that the body would move along the diagonal which is the vector sum of the motions produced by the forces acting independently.

Varignon arrived at the same conclusion, but he employed his principle of moments to deduce the same result.

Stevinus made other contributions to the Science of Mechanics. He rediscovered, by methods entirely his own, some of the principles which Archimedes had previously formulated.

Stevinus' Work in Hydrostatics.—Stevinus also introduced two ideas which have been of great assistance both to him and to later investigators in their work on hydrostatics. The first of these was a sort of endless-chain concept of fluid behavior (if a given volume of fluid goes down, an equal volume of fluid must go up). The other idea was the assumption that, when a small portion of fluid is assumed to be solidified, this does not disturb its state of equilibrium. With these tools he demonstrated that the hydrostatic pressure is independent of the shape of the containing vessel and depends only on the height or depth of the fluid and on the area of the base of the column of fluid.

Summary of Work of Stevinus.—Stevinus also formulated the conditions for equilibrium of three forces acting in a single plane and passing through a common point (coplanar concurrent forces). It is likewise apparent that he recognized a distinction between statical and dynamical problems because he states that finding the magnitude of a force which will support a loaded wagon on an inclined plane is a simple statical problem while finding the force which will move the wagon up the plane requires additional considerations.

Stevinus' writings indicate that he possessed some idea of the principle of virtual displacements. He states, "What a simple machine gains in force, it loses in distance." Also in his work on pulleys he says: "As the space passed over by the force is to the space passed over by the resistance, so is the resisting force to the applied force." This is approximately the principle of work; but, as previously noted, he does not formulate it into a definite statement of principle.

The important contributions of Stevinus to the development of the Science of Mechanics may therefore be stated as:

(1) Demonstration of the parallelogram of forces
(2) Demonstration of the principle of the inclined plane
(3) Exposition of buoyancy

Because of his work on the inclined plane and the parallelogram of forces and his application of these tools to the solution of more extended problems, Stevinus is generally credited with being the originator of the Science of Graphical Statics.

Basic Statics Completed.—While the pioneers in all fields of scientific development are today being highly honored, many people do not realize how different the task of the pioneer was from that of those who have followed along the well marked paths which were blazed for them by these early hunters for scientific knowledge. As we look back on what we now consider the simple facts or principles which these men have demonstrated or formulated, the mental capacity which was required for the task very often is not truly evaluated. A pioneer must have many qualities not required of those who follow him. He must be a man of great physical and mental courage and integrity of purpose; he must possess the ability to visualize and speculate about what is ahead as well as unusual talent in his particular field.

Science in general owes its origin to the development and accumulation of a sufficiently large number of facts and principles over the years to require that these facts and principles be digested and put into a form which can be easily communicated to others. This formulation or classification of knowledge often brought out additional knowledge not discovered by the original investigator. An example is Archimedes' statement of the principle of the lever, wherein he uses the product of the weight and the length of the lever arm; while years afterward the correct product was shown

by da Vinci and also by Ubaldi (1545–1607), in his book *Mechanicorum Liber* (1577), to be the product of the weight and the perpendicular distance from the fulcrum or the statical moment. This book by Ubaldi also discusses the pulley, the wedge, and the screw.

Instinctive methods have been of great help in the development of science, but they have also led to some serious errors, as has been pointed out earlier.

With the establishment of the principle of the inclined plane by Stevinus, the laws of equilibrium for all simple machines were easily arrived at by the simple process of deduction. Now that the basic principles of simple statics had been established, all that remained to be accomplished was the development of a satisfactory workable technique for the application of these principles to the individual problem. This, in itself, seems a simple matter, but examination of a liberal sample of books written since the time of Stevinus will convince the reader that even the development of this operational technique was not something which was easily conceived but was a skill which required hundreds of years of effort of the best analytical minds of the world.

At this time it may be well to point out that the development of these principles, which today the college sophomore or junior is expected to absorb in his first few weeks of study of Mechanics, took the super-intellects of these pioneers of science just about 2000 years to discover and to digest into what today would be considered a very crude and unsatisfactory form. Approximately another 500 years were needed to arrive at our present-day streamlined technique. Thus, Statics in its present state is the result of about 2500 years of development by many exceedingly capable minds. It may be that we modern teachers of Mechanics should have more sympathy for our students when they have trouble grasping the basic concepts of these simple—to us—but important and far reaching principles of Statics.

CHAPTER 6

Dynamics, the Science of Motion

Early Theories of Motion.—Even the early Greeks were somewhat aware that bodies at rest and in motion presented different problems. In just what manner the two situations differed they were unable to determine, but with their usual optimism they were perfectly willing to attempt an explanation of the behavior of moving bodies. They did not realize that their speculative methods were entirely unsuitable for the study of such problems and could only result in confused and unsound theories which, instead of simplifying and clarifying, only led to greater involvement.

The Greeks at this early period had no satisfactory or accurate means of measuring distance or time, which of course were the primary factors to be considered in any study of moving bodies. Under such conditions as these a confusion of ideas was inevitable.

Aristotle divided motion into two types, "natural" and "violent." Other investigators tried to sub-divide "natural" motion into what they called "voluntary" motion (uniform circular motion) and "natural" motion which is stronger or more rapid at the end (freely falling bodies). "Violent" motion was supposed to include all other types of motion.

All these early workers made one serious error. They assumed that force was necessary to maintain motion and not simply to overcome resistance to motion or change its direction. This error caused them to speculate about the origin of the forces necessary to keep the planets moving, and led them into a variety of errors and false assumptions.

No sound experimental approach to the solution of the problem of motion was made until the work of Copernicus

gave Galileo the courage and incentive to start his career as an investigator of the primary laws which govern motion.

Galileo, Creator of Modern Science.—The life of Galileo (1564–1642), who has sometimes been called the creator of modern science, is crowded with events and accomplishments the story of which should be read by every person interested in the great men of the world. Galileo's trials and tribulations and his contributions to science should be part of the knowledge of all scientific students and workers of the present day. They would then better appreciate the wonderful heritage passed on to them by their many brave and self-sacrificing predecessors in the quest for scientific knowledge. Our modern scientific investigators suffer little of the religious and political persecution, and expose themselves to none of the personal dangers, which such men as Galileo and others of his time faced in order that science might advance and become what it is today—one of the most powerful tools in the development of the human race.

Galileo, the eldest son of a Florentine nobleman, was born at Pisa on February 18, 1564. There were five other children, two boys and three girls. Like many great thinkers, he early developed ability in constructing and experimenting with mechanical gadgets and instruments.

Even though he was of noble birth, lack of money always seemed to be a factor which influenced the pattern of Galileo's life. During his boyhood Galileo developed a great interest in music, drawing, and painting, and he devoted much of his time to the study of these arts. He wanted to become a painter, but his father apparently thought medicine was a more lucrative field and sent him to the University of Pisa in 1581. Here he studied medicine under the great botanist, Andrew Cae, but continued his study of the arts also. He was soon convinced that, if he were to continue to progress in art and music, he would also have to study mathe-

matics. His work in geometry was carried on under the direction of Ostillio Ricci.

This study of geometry was a most important milestone in Galileo's life because the reading of Euclid so stimulated his interest in mathematics that he was able to overcome parental objections and devote his entire energy to the study of this science. Thus, he was started on the path which he continued to follow during his productive life.

After Euclid he proceeded to investigate the writings of Archimedes and became so interested in the famous bathtub incident that he wrote a paper explaining the hydrostatic principles which enabled Archimedes to discover the method of deception used by the jeweler who had made the gold crown for Hieron and failed to return some of the gold supplied for the purpose. The writing of this paper, while its content was not of profound scientific importance, proved to be another milestone in Galileo's development because it attracted the attention of Guido Ubaldi, a distinguished mathematician. Ubaldi became interested in the young man and in 1589 secured for him an appointment as lecturer at Pisa. Thus, at the age of twenty-five, Galileo was well started in his career in mathematics and science.

Like many men with keen minds, Galileo was impetuous and high spirited and at times lacked judgment in dealing with people. He was not discreet in expressing his opinions of the work of other thinkers. His study of Aristotle had caused him to doubt some of the theories there presented and he did not hesitate to express his opinions about them.

Galileo Experiments With Falling Bodies.—Soon after taking the position at Pisa Galileo became involved with the followers of Aristotle over the theory of falling bodies. The Aristotelian theory was that the velocity of a falling body was dependent on the weight of the body, heavy bodies falling faster than light bodies. Galileo maintained that the velocity

of free fall was entirely independent of the weight. To demonstrate the correctness of his theory he ascended the leaning tower of Pisa and at an elevation of 100 feet released together a 1-pound ball and a 100-pound ball of the same size. Both balls reached the ground at the same instant. His critics were not convinced, however. They maintained that some occult influence controlled the behavior of the balls. Galileo demonstrated also that on smooth inclined planes the velocity is independent of the slope and depends only on the change in elevation of the moving object. This also was contrary to the teachings of Aristotle.

While at Pisa, in 1590, Galileo wrote a series of lectures on the motions of bodies. These lectures remained in manuscript form until they were brought together by Alberi and published in fifteen volumes at Florence (1842–1856) as *Works of Galileo*. It appears that these lectures were used by Galileo as the base for his *Discorsi e Dimostrazioni Matematiche*, or *Dialogues Concerning Two New Sciences*, which was published in 1638.

Galileo soon realized that to question the philosophy of Aristotle was to make himself exceedingly unpopular with the dignitaries of the Church who at this period were the all-powerful group. The Church had established a rigid system of dogmas. It considered scientific progress neither necessary nor desirable. It decided that the Aristotelian philosophy should be accepted as the unquestioned truth and that any who disagreed with this edict were to be considered enemies of the Church.

Galileo, being a man of vision and anxious to learn the true scientific facts, would not and could not accept such a philosophy with mental integrity. He soon became *persona non grata* to the Church, and there was much intrigue against him. In addition, he got into trouble with an influential citizen by the name of Giovanni de Medici over the construction of a machine being built to clean the harbor of

Leghorn. By this time his position at Pisa became so difficult that he resigned and returned to Florence in 1591.

He was then offered the chair of mathematics at Padua with a six-year appointment at an increased salary. This he accepted in 1592. His lectures on the laws of motion, fortifications, astronomy, and mechanics soon made Galileo famous. However, shortly after going to Padua his father died and money—or the lack of it—again began to influence his life. He was forced to support the family and, to do this, he had to devote much time to tutoring, which greatly decreased the time and energy available for his research projects.

After his first term at Padua was completed he was reappointed for another six years. At this time he was accused of living illicitly with a certain Marina Gamba, who bore him two daughters and a son between 1599 and 1610. However, the Senate did not consider this incident in his private life important enough to investigate, and reappointed him at an increased salary.

Galileo Becomes Interested in Astronomy.—During his stay at Padua, Galileo developed a friendship with Kepler (1571–1630), who was born at Weil, Wurtemberg, and was educated at the University of Tübingen. Their interest in astronomy furnished the common meeting ground. Kepler sent Galileo a copy of his book *Prodromus Dissertation Cosmographicarum seu Mysterium Cosmographicum* (1596). In the resulting discussion Galileo told Kepler that for many years he had favored the Copernican system over the old Ptolemaic system and that he had discovered many arguments in favor of the newer Copernican theory. Kepler tried to persuade Galileo to publish his opinions on the relative merits of the two systems but his experience at Pisa had taught Galileo something about human psychology and he realized that any publication which expressed his opinion on the subject would only start a violent wave of criticism against him.

Soon after his reappointment at Padua, Galileo gave a course of lectures on a new star which appeared in 1604. According to the Aristotelian philosophy the heavens were complete and, therefore, no new stars could appear. Here again Galileo had started a new controversy between the followers of the two astronomical systems, and he was now forced to defend the Copernican theory against the old Ptolemaic system. This was a difficult situation for the believers in the old Aristotelian philosophy to deal with, for here was an opponent of great wisdom and prestige.

At about this time Galileo learned of an invention by a Dutch optician, Hans Lippershey, which brought distant objects closer to the observer. It was not long until Galileo had constructed a telescope with a power of three, and he soon increased this to thirty. Now, equipped with the tools which permitted efficient and minute examination of the heavens, he was able to learn much about the planetary system by direct observation; and he soon found many bodies which, according to the old Ptolemaic theory, could not have existed. There were still many skeptics, however, and many of those who were regarded as savants of the time refused to look at the heavens through the telescope.

Soon after Galileo had come to Padua a Venetian, Giordano Bruno, was brought before the Inquisition, was convicted, and was burned at the stake for heresy. His support of the Copernican theory was one of the charges against him. This evidence of the antagonism of the Church to the progress of scientific knowledge must have caused Galileo considerable anxiety and no doubt tended to dampen his zeal for the discovery and development of the principles of science. He continued his study of the heavens, and soon concluded that "man's home in space is only one of a number of small bodies revolving around a hugh central sun." To protect himself against charges of heresy he published his findings in the form of anagrams.

Galileo's Defense of the Copernican Principles Incites Criticism.—After eighteen years at Padua, Galileo was invited by the Grand Duke of Tuscany to return to Florence. In making this change of positions Galileo showed that he was a business man, as well as a scientist, as he obtained a salary increase from 520 to 1000 florins. In addition his contract specified no definite duties but permitted him to devote practically all of his energy to study and writing. There was, however, one phase of the situation which Galileo had failed to consider. Tuscany was a Papal State and thus came under the direct jurisdiction of Rome (Pope Paulus V) and the Vatican. Soon after taking this new position, he began to feel the displeasure of the Church and the Aristotelian Jesuits. This displeasure was largely due to his championship of the Copernican theory and his use of this theory to explain certain hitherto puzzling Biblical statements.

Some of the most ridiculous arguments used against Galileo were developed by one Francisco Sizzi. He said there had to be seven planets, and only seven, because there were seven windows in a man's skull—two eyes, two nostrils, two ears, and a mouth; also, since there were seven days in a week and each had been named after a planet, there could be no more planets without disrupting the week, which he seems to have forgotten was a man-made device.

In 1611 Galileo went to Rome and was highly honored by the State and Church, despite the many enemies his radical theories had made for him.

In 1612 he published a paper on floating bodies which roused considerable new antagonism, and he had much trouble defending his ideas.

Galileo was the leading scientist of the day, with plenty of money and time to devote to his theories. Proud and haughty now, he replied to his critics with ridicule and sarcasm. This, of course, made him even less popular. Finally he was called to Rome to justify his teachings and beliefs. During this

hearing he helped his opponents to state their case and then promptly proceeded to refute everything they had built up by the sheer logic of his own arguments. After this examination he went back to Florence, where he continued to advocate the Copernican system until the opposition grew so strong that he was finally forbidden by the Church to teach or believe in the motion of the earth.

In 1623 the Pope died and Cardinal Barberini became Pope Urbanus VIII. Galileo returned to Rome and had many audiences with Urbanus, who seemed to be favorable to him. This so encouraged Galileo that he returned to Florence and in 1630 completed his *Dialogues on the Ptolemaic and Copernican Systems* (*Dialogo dei massimi sistemi del mundo*). This book consisted of conversations between three characters on the two systems. In it considerable ridicule and sarcasm was directed against the many absurd statements made by the advocates of the Ptolemaic system. Galileo had great difficulty in getting the book published; but, after two years (1632), he succeeded and it soon was greatly sought after, even though his churchly opponents immediately tried to suppress it.

His opponents grew so strong that they were able to prevail on the Pope to call Galileo before the Inquisition for trial as a heretic. During the progress of the trial Galileo was treated with the respect which only one of his ability and mental stature could command. He was convicted and made to recant, but no physical penalty was imposed. However, he was under surveillance and virtually a prisoner for the rest of his life. He died in 1642, at the age of seventy-eight, a broken man.

Study of the history of heresy shows that unorthodox doctrines were sometimes freely presented by some individuals, while they quickly brought death to others. The verdict seems to have depended not so much on how radical the opinions or theories were as on the manner in which they

were presented. Ridicule and contempt for what is sincerely believed to be right by any group is bound to cause reprisals.

No individual or group of individuals can hope arbitrarily to control or dominate the thoughts or opinions of other individuals or groups. But they may, as we all know, strongly influence or direct the flow of thought by a judicious and diplomatic process of education which may have to extend over a considerable period of time to accomplish the desired result successfully. This result, however, must have a logically sound base if it is to continue to maintain its prestige indefinitely.

In his *Perennial Philosophy*, Aldous Huxley says: "Those who choose the profession of artist, philosopher, or man of science, choose, in many cases, a life of poverty and unrewarded hard work. But these are by no means the only mortifications they have to undertake. When he looks at the world, the artist must deny his ordinary human tendency to think of things in utilitarian, self-regarding terms. Similarly, the critical philosopher must mortify his common sense, while the research worker must steadfastly resist the temptations to over-simplify and think conventionally, and must make himself docile to the leadings of mysterious Fact. And what is true of the creators of aesthetic and intellectual goods, is also true of the enjoyers of such goods, when created. That these mortifications are by no means trifling has been shown again and again in the course of history. One thinks, for example, of the intellectually mortified Socrates and the hemlock with which his unmortified compatriots rewarded him. One thinks of the heroic efforts that had to be made by Galileo and his contemporaries to break with the Aristotelian convention of thought, and the no less heroic efforts that have to be made today by any scientist who believes that there is more in the universe than can be discovered by employing the time hallowed recipes of Descartes. Such mortifications have their reward in a state of conscious-

ness that corresponds, on a lower level, to spiritual beatitude. The artist—and the philosopher and the man of science are also artists—knows the bliss of aesthetic contemplation, discovery and non-attached possession."

Galileo's Experimental Science Challenges Established Theories.—We have examined the accomplishments of the ancient Greeks and the methods by which they formulated their theories. Also the Church—the dominating power in the world for centuries—was shown to have been a retarding, rather than a stimulating, influence to scientific development for almost 1500 years. These things tended to work together to the detriment of scientific progress. They discouraged any progressive thought, with the result that little or no actual progress was made from before the birth of the Christian religion until Copernicus produced his new theory of the planetary system. Even this theory was successfully smothered for many years, but eventually the forces of intellectual integrity began to grow stronger and were able to overcome the intrigue of the demagogues of the Church.

What the world needed was a master intellect with the courage and will to challenge the long established theories—a crusader for fundamental theory based on experimentally proven knowledge. When such a leader appeared, scientific progress again took up its forward march. Galileo was the man who started this new quest for true scientific knowledge and gave birth to the true experimental science wherein facts were acquired by observation and experiment and were accepted as found without any attempt being made to make them part of a previously developed philosophy evolved by previous thinkers without factual evidence. Unlike the Greeks, Galileo was interested in the "how," not the "why," of things. Another important factor in the new approach was that to Aristotle time and space were unimportant while to Galileo they were primary and fundamental concepts.

Galileo was the first man to have the courage to step forward and challenge the validity of such Aristotelian fallacies as are presented in the following statements:

(1) Substances are divided into corruptible and incorruptible.
(2) Bodies are classified as absolute heavy bodies and absolute light bodies and seek their places, the light bodies on top.
(3) Motions are classified as natural motions and violent motions.
(4) Large bodies fall quicker than small ones.

Such visionary and incompetent thinking could not possibly satisfy or convince a man with Galileo's urge to uncover true scientific principles. Had the early investigators possessed a better conception of what their actual task involved, they would have confined their efforts to the development of the basic theory of Statics, leaving the more difficult problem of the behavior of moving bodies for later consideration. By the time the first problem had been solved, they would no doubt have realized that time was an exceedingly important concept of the second problem. This realization then would have brought about an earlier development of some sort of satisfactory device for the measurement of time. There was no accurate means of measuring time until Christian Huygens (1629–1695) developed the first successful pendulum clock.

The Period of the Simple Pendulum.—Galileo's interest in things mechanical was demonstrated early in his career. Shortly after he entered the University of Pisa at the age of seventeen, he observed a large lamp swinging back and forth. Having no timing device, he proceeded to estimate the time of swing by counting his heart beats. With this crude timing he concluded that the time of any given swing was independent of the amplitude. Further observation also led to the

conclusion that the time of the swing was a function of the length of the supporting chain and not of the amplitude. This law of pendulum operation seems to have been Galileo's first real contribution to the Science of Mechanics. It proved to be a much more important discovery than it might seem, since it was used shortly afterward by Huygens in the development of his clock.

Laws of Free-Fall.—Early in his career as a professor at Pisa, Galileo became interested in the commonly believed theory that heavy bodies fell faster than light ones. His study of this theory brought about the famous leaning tower of Pisa experiments (1590) referred to previously. As the result of his early observations he concluded that the velocity of fall varied directly with the distance, but he soon learned that this conclusion was incorrect. Later experiments showed him that the velocity varied directly with the time of fall. He was able also to prove this by logic.

Here we see an important difference between Galileo's methods and those of the early Greeks. As has been pointed out, they rarely attempted to check their theories by experimentation. Galileo did, and he was often able to convince himself that his logic was incorrect.

Since he had no clock, he had to construct a mechanism which would serve to measure time of fall during his experiments. This he did by building a tank with a large cross-sectional area so that when water was permitted to flow out through a small hole in the bottom of the tank there was little change of head on the orifice during the time of flow. With this device he was able to determine time quite accurately by measuring the quantity of water discharged during the period of fall of the weight. After having established that the velocity varied with the time of fall, he soon learned that the distance traveled by the falling weight varied with the square of the time of fall.

Motion on the Inclined Plane.—After he had determined the laws of free-fall, the next logical step in his investigations was to compare free-fall with motion down an inclined plane. Here he proved that the final velocities were the same for any given change in elevation; also that the times of travel between two elevations by free-fall and down an inclined plane were proportional to the distances traveled by the moving object.

In his *Discorsi e Dimostrazioni Matematiche*, published in 1638 and translated into English under the title *Dialogues Concerning Two New Sciences* by Crew and Salvio in 1914, Galileo also demonstrated that the motion of a simple pendulum was comparable to the motion of bodies on inclined planes in that, if free to do so, the weight in either case would always return to the elevation from which it started. To demonstrate this, he constructed a pendulum with a cord of variable length, as shown in Fig. 9. If the weight starts at A

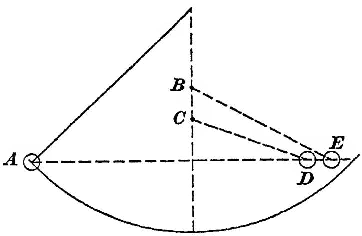

FIG. 9

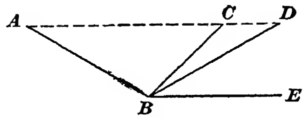

FIG. 10

and swings until the cord strikes a pin at B or C, it will return to the elevation of A at a point D or E, the position depending on which pin is in place. In Fig. 10, if the weight starts from A and slides down a smooth plane to B and then moves up either smooth plane BC or BD, it again returns to the elevation of the starting point A, just as the pendulum weight did. Galileo saw that these two cases had something in common, but he did not formulate any principle of work.

The world had to wait for some time before that principle was discovered.

From his study of the motions on the inclined planes in Fig. 10 he decided that, since there was less resistance in going up the plane *BD* than in going up *BC*, the body could travel a greater distance up plane *BD*. It then followed that for the limiting case of the horizontal plane *BE*, with no resistance to motion, the motion should continue indefinitely and at a constant undiminishing speed. This was a demonstration of the now well-known law of inertia. A body in motion and not acted upon by any external forces in the direction of motion continues to move indefinitely in a straight line at a constant velocity. Galileo probably did not realize the importance of this demonstration. It remained for Huygens (1629–1695) and Newton (1642–1727) to formulate the principle into a law and to put the principle into use.

Principle of Continuity.—In making the investigations of free-fall and motion on the inclined plane Galileo developed a method of approach which probably has been more valuable to the world than his actual discovery of these basic laws of Mechanics. It has been called the *principle of continuity* and is what we moderns call the *scientific method.* When he was attempting to develop a theory, he proceeded to modify one condition at a time as far as possible or until he had established the limiting conditions for the variable, as has just been illustrated in his experiments on the inclined plane.

The importance of the discovery of such a technique cannot be overestimated. It has become one of the most valuable tools of our present-day scientific workers in all fields and has lighted the path to final success for many of our important scientific discoveries, which no doubt would have been long delayed or maybe never uncovered without this technique.

Probably the greatest benefit to be reaped from the study of the historical development of any science is the discovery of the methods of investigation employed by the successful workers of the past, or the discovery of "the how" of their work rather than "the what." Such knowledge usually proves not only useful but stimulating to the workers who follow the pioneers.

Path of a Projectile.—After his work on the inclined plane, Galileo turned his attention to the problem of the flight of a projectile. Since investigators had started to question the theories of Aristotle regarding violent and natural motion, they began to realize that change of motion was caused or brought about by external reasons, not by any property of the motion itself.

Through his experiments with bodies on inclined planes, Galileo had learned that any change in the motion was always due to some change in the externally applied conditions, such as a change in the slope of the plane or the surface condition thereof. He had also established that, if there were no such external factors involved, as when the block moved on a perfectly smooth horizontal plane, then the motion would continue indefinitely as in the case of the block on plane *BE* in Fig. 10.

It is not definitely known who was the first to put the first law of motion into its finally accepted form, but Galileo in his *Discorsi* gives a correct statement of the law as follows:

"Conceive a movable body upon a horizontal plane and suppose all obstacles to motion to be removed. It is then manifest from what has been said more at large in another place that the body's motion will be uniform and perpetual upon the plane, if the plane is indefinitely extended."

This statement of the law says that all external obstacles or forces resistant to motion are to be removed, but absence of resistance is impossible under any ordinary practical

conditions. We do not have frictionless planes or a vacuum for bodies to move in. Finally, investigators began to realize that all moving bodies with which man has had actual experience are influenced or acted upon by some sort of resisting or propelling forces. There are no examples of free and unrestricted motion available for our study and observation.

Because this principle—which has long been known as the First Law of Motion—was unknown to the early investigators, many and various attempts were made to explain the change of velocity of falling bodies and the motion of projectiles.

In 1561 Santbach proposed the theory that a projectile traveled in a straight line until its velocity in that direction became zero, and it then fell vertically to the ground. He presented a treatise on gunnery which was based on this assumption. Later he modified this theory to the assumption that the projectile descended by steps, not along a curve. Others such as Tartaglia (in *La Nova Scienta*, 1537) thought that the path was first a straight line, which was followed by the arc of a circle. Tartaglia also stated that a 45-degree firing angle gave the greatest horizontal range, but offered no proof of this statement. To us of the present generation such assumptions seem unworthy of men with any scientific ability at all. These men seem to have obtained the impression that during the early part of the flight the horizontal velocity was so great that gravity did not act. Such an illusion is not so absurd when we think of a stream of water flowing from a horizontal nozzle. It appears to travel a considerable distance in the horizontal direction before it starts to fall. The extremely short time which is required for the water to travel over the apparently horizontal part of the path is disregarded. However, it must be remembered that the early investigators were living in the dark, as it were. They had none of our modern scientific equipment, such as high-speed cameras and accurate time-measuring instru-

ments, with which to make observations. All they could do was guess. That such guesses were often wrong is not at all surprising. We should not be too critical of the degree of accuracy obtained. All we need to do to retain our sense of humility is to examine the validity of the deductions of some of our present-day forecasters, even when equipped with all the tools and information of their trade.

In his second "Dialogue," Galileo made Simplicus say, "Since there is nothing to support the body when it quits that which projects it, it cannot be but that the proper gravity must operate." In this manner Galileo stated his solution and then proceeded to explain how he arrived at this conclusion. "Bodies will fall in the most simple way, because natural motions are always the most simple. When a stone falls, if we consider the matter attentively, we find there is no addition, no increase on the velocity more simple than that which is always added in the same manner." The weakness of this explanation is that it is quite often difficult to discover what is the easiest or simplest path for nature to follow. Galileo himself was a victim of this deception, when he first assumed that the velocity of a falling body was proportional to the distance through which it had fallen instead of proportional to the time of fall. A little later in his discussion he makes the statement, "The cause of the acceleration of motions is not a necessary part of their investigation." This was evidently an evasion by Galileo because he realized that he had no satisfactory solution to offer for this phase of the problem. The action of gravity was not to be understood or clearly explained until years later.

The Independence of Action of Forces.—Galileo thought gravity was a constant uniform force. Descartes (1596–1650) did not agree with this theory, as is indicated by the following statement[1] of his: "It is certain that a stone is not

[1] *History of Inductive Sciences*, by Whewell, p. 328.

equally disposed to receive a new motion or increase of velocity when it is already moving very quickly and when it is moving slowly."

When Galileo stated that gravity began to act on the projectile as soon as it left the supporting device, he not only solved that problem but also refuted for all time the theory that a body could be acted upon by only one force at any given time. This recognition by Galileo of the independence of action of two or more forces which act on a body at the same time was an exceedingly important discovery. It opened the way for many basic developments, not only in Mechanics but also in other branches of science that involve in their operation both the composition or synthesis of effects and the analysis of a resultant effect into its component parts. A few examples of these would be the parallelogram method of determining a resultant force, the composition of light waves, and the transfer of thermal energy from different sources to a central object, or the reverse of these operations. Having shown the presence of the two forces, gravity and the horizontal projecting force, Galileo then proceeded to demonstrate that the path of the projectile was nearly a parabola.

Curvilinear Motion.—Galileo's solution of the projectile problem was also the solution of the problem of curvilinear motion. Even the Greeks had known of the existence of the transverse force when a weight is revolved at the end of a cord. Also G. B. Benedetti in 1585 had published *Diversarium Speculationum Mathematicarum et Physicarum* at Turin, in which he stated that a body acted upon by a centrifugal force would, when released, travel in the direction of the tangent rather than along a radius as had been the belief up to that time. However, the determination of the curved path was not solved until Galileo in his *Discorsi* explained the motion of the projectile by stating what is essentially the Second

Law of Motion. He states that a body projected horizontally will have added to this movement a uniformly accelerated motion downwards (that is, the motion of a body falling vertically from rest), and the composite effect will therefore be that the body moves along a parabolic path. This statement apparently was not understood at the time by other investigators. Galileo then attempted to explain that the theoretical results did not agree with the experimental ones because of the excessive velocities of projectiles which he called "supernatural velocities." He apparently was not aware of the resistance offered by the air.

With the theory of uniformly accelerating forces developed, as when Galileo demonstrated the theory of falling bodies, the only difficulty remaining for the solution of problems involving variable forces was the lack of a mathematical technique for handling such forces. This solution had to await the development of the calculus, which was not to come for some years.

Galileo's Contributions to Mechanics.—The discoveries of Galileo in the field of Mechanics may be summed up by the following:

(1) All bodies fall the same distance in the same time (air resistance neglected).
(2) The velocity of a falling body is proportional to the time.
(3) Distance of fall is proportional to the square of the time.
(4) Actions of simultaneous forces are independent.
(5) There is an invariable relation between mechanical cause and effect.
(6) Force is a mechanical agent, not a property of matter.
(7) In raising a weight with a lever, we lose in speed what we gain in force.

While this list of discoveries is indeed significant and important, it is the opinion of this writer that Galileo's

greatest contribution to Mechanics and mankind probably lay in his personality. He really was the man who dared, in the face of great personal danger, to shake the world out of its self-satisfied state of "there could be nothing new" into a realization that "there still is much to be learned." For this awakening and stimulation of the minds of men the world of today owes Galileo and his predecessor Copernicus a debt of incomprehensible magnitude. They were the men responsible for the awakening which made possible our present-day scientific development.

In his *Mécanique Analytique*, published in 1788, Lagrange credits Galileo with being the originator of the modern method of scientific investigation. "Dynamics is the science of forces accelerated or retarded and of the various movements which these forces can produce. This science is due entirely to the moderns, and Galileo is the one who laid its foundations. Before him philosophers considered the forces which act on bodies in a state of equilibrium only; and, although they could only attribute in a vague way the acceleration of heavy bodies and the curvilinear movement of projectiles to the constant action of gravity, nobody had yet succeeded in determining the laws of these daily phenomena on the basis of a cause so simple. Galileo made the first important steps, and they opened a way, new and immense, to the advancement of Mechanics as a science."

While Galileo's most important contributions to the Science of Mechanics resulted from his studies of motion, he was also the first to study what we of the present generation most often call Resistance of Materials or Mechanics of Materials.

The common assumption during Galileo's time was that, if two structures were built of the same material and there existed a constant ratio of the size of all parts, then the strength or ability to resist external forces was directly proportional to this ratio. Galileo did not agree with this

theory and in rebuttal called attention to several examples which were common knowledge to everyone. One of these was that a horse falling a few feet would break its bones while a dog would be unharmed. Another was that a small obelisk or column could be laid on its side without injury while a large one would fail (crack or break) because of its own weight. This latter failure was demonstrated when a large column was laid on the ground and supported at the ends. Later it was thought advisable to support the column at the center. A short time after the third support was added, it was found that the column had broken. Investigation showed that one of the end supports had settled, leaving the entire weight of the column on the other two. If the column had been smaller, but of the same relative proportions and material, it would not have failed.

This last case was the beginning of the theory of beams. In his further discussion of beams, however, Galileo failed to realize that at any section of the beam there was equilibrium of the tensile and compressive stresses. This equilibrium of the tensile and compressive stresses was not discovered until years later, by E. Mariotte in 1686 at Paris. Galileo also demonstrated that for a given weight per unit length a hollow cylinder is stronger than a solid cylinder. Galileo arrived at his conclusions about strength of materials largely because of his observations of the works of nature, since he had no apparatus for making physical tests on actual specimens or models.

CHAPTER 7

Period Between Galileo and Newton

After Galileo there was a period of approximately 100 years during which, as is usually the case when a great scientific thinker passes on, a number of less prominent yet important contributing names appear in the records of the Science of Mechanics. This has been the history of the development of every branch of science. Whenever an outstanding man appears, he is followed by a group of lesser satellites who probably draw their inspirations from the work of the leader and proceed to expand and clarify the application of his fundamental ideas and principles.

During this particular period we find the names of Kepler (1571–1630), Francis Bacon (1561–1626), Marcus Marci (1595–1667), Descartes (1596–1650), Boyle (1627–1691), Pascal (1623–1662), and Huygens (1629–1695), whose contributions to the Science of Mechanics were of this nature.

Descartes and Analytical Geometry.—Of the group just mentioned probably the most talented was René Descartes (1596–1650), who was born at LaHaye (near Tours), France. His application of algebra to geometry—which is the basis of analytical geometry—is generally considered the most important mathematical development of the seventeenth century.

Like many of the men of great ability, he had a wealthy father, who owned much land, and Descartes received all the advantages that money could provide. Descartes was not of those who believe that genius thrives best on hardship. He was convinced that good mental work could only be produced when conditions were ideal; so he limited his working day to just a few hours and devoted the rest of his time to society and goodfellowship. Such a philosophy seems to have largely

disappeared from the plan of life of those who are generally recognized as the successful men of today. This result is no doubt due to the exceedingly more complicated and involved nature of our existence, which turns the modern specimen of the *genus-homo* into a machine devoted almost entirely to the process of creating gadgets and implements for making life still more complicated in order that he may thus furnish other men with more opportunities for doing the same thing.

If we examine the human products of our twentieth-century living, we can well wonder if our progress in human relations has not been left far behind in our zeal for the development of scientific knowledge. Maybe we shall have to produce a Galileo of human relations and living before this modern world again becomes a satisfactory place for good living.

Descartes was an independent person who was not particularly interested in the work of others. He traveled extensively but did not make any effort to visit his fellow scientific workers, even going to Florence without making any contact with Galileo, who certainly would have roused curiosity in most men. Further evidence of his independence was shown when he became the first to discard Latin in publishing his work.

Descartes' development of analytical geometry was exceedingly important from the standpoint of Mechanics because without it Newton could not have produced his significant *Principia* (1687) or made his other important contributions.

Descartes was much influenced by Galileo's troubles with the Church. This influence no doubt materially restricted his contributions to science for at one time he threatened to destroy all his writing because of his fear of incurring the displeasure of the Church. He did not do this, however, but he delayed publication for about ten years.

Even then his books were placed on the *Index Expurgatorius*. He died at the early age of fifty-four.

Huygens, the Dutch Archimedes.—Christian Huygens (1629–1695), often called the Dutch Archimedes, was born at The Hague and educated at Leyden and Breda. He was another of our great thinkers who chose his parents well, thereby avoiding much of the difficulty experienced by less fortunate individuals. He came of an old artistocratic Dutch family of wealth and distinction. His father, Sir Constantijn Huygens, was a leader of Dutch culture and had decided on a literary and diplomatic career for his son. This was not followed because Huygens early in life exhibited great aptitude for mathematics and science, indicating his prodigious mental power and vitality. At the age of twenty-two his mathematical ability attracted the attention of René Descartes who saw great possibilities ahead for him. His later career fully substantiated Descartes' faith. Huygens is considered by many the peer of Galileo and in geometrical ability his superior. His enthusiasm never left him; even as an aged man he produced new ideas and inventions.

Huygens made important contributions to the sciences of mathematics, astronomy, optics, and physics, in addition to his fundamental work in Mechanics. He probably is most famous for his wave theory of light propagation. This theory was first put in writing about 1678, but it was not published in its final form until 1690 when it appeared as *Traité de la Lumiere*. This theory of light propagation was strongly opposed by Newton, who presented what is known as the corpuscular theory. Newton maintained that light was not transmitted by waves through the ether but was propagated by a stream of minute particles traveling in straight lines from the source of light, much as an arrow might fly. Huygens' wave theory was finally established 100 years later by Dr. Thomas Young.

In mathematics Huygens was the pioneer important writer on the theory of probability. He published a pamphlet on the subject in 1657 at Leyden. This was the first attempt to organize the available material on a subject which had been introduced by Chevalier de Mere, a professional gambler, 400 years before.

The Compound Pendulum, the Center of Oscillation, and the Principle of Work.—Huygens' most important contribution to the development of Mechanics was his work on center of oscillation and the compound pendulum, which he presented in his book *Horologium Oscillatorium* (1673) and from which Newton obtained much for his *Principia* (1687). Huygens was also the first to solve problems involving the dynamics of several masses.

Descartes had also done considerable work on the pendulum, but Huygens was the first to develop a general solution. His theory was that a compound pendulum should behave in a manner such as would be expected from a lot of simple pendulums if fastened together. The shorter ones would be retarded and the larger ones accelerated by the others. The entire group or a compound pendulum would therefore move or act as a simple pendulum of some intermediate length. The length of this simple pendulum would be the same as the distance from the center of oscillation to the center of rotation. Huygens arrived at this deduction from the principle of work. He decided that when the pendulum was set in motion the center of gravity of the mass could not rise higher than the elevation from which it started; otherwise, heavy bodies could be caused to raise themselves to any desired height. This fact was known to many, but no one had presented it as a fundamental principle of Mechanics. In the course of his mathematical development of the theory of the compound pendulum, he measured work in terms of $\frac{1}{2}Mv^2$ which he called *vis viva*.

Huygens concluded that in a frictionless system the distance which the center of gravity fell would always be equal to the distance to which it could rise.

During the process of his mathematical development of the theory of the compound pendulum, Huygens encountered the term ΣMr^2; but he did not give it a name. It was first called "moment of inertia" by Euler (1707–1783). Huygens also was the first to use the transfer formula for moments of inertia. Other contributions by him to the Science of Mechanics were the determination of the acceleration of gravity by the pendulum method and the explanation of uniform circular motion in which he demonstrated that the value of the normal acceleration was $\frac{v^2}{r}$.

In connection with his work on the pendulum, Huygens maintained that the term $\frac{1}{2}Mv^2$ represented the efficacy (the power to produce effects) of any moving body. With this Descartes disagreed and maintained that Mv was the correct term. This difference of opinion between the two men started a controversy which lasted many years, until D'Alembert in his *Traité Dynamique* (1743) showed that both quantities were correct but that they did not represent the same thing.

In retrospect we thus see that, while Huygens made many valuable contributions to science in general, his work in Mechanics was not so vitally important to the future development of the science as that done by some of the other early contributors. The science could have been developed without Huygens' contributions. This fact will be discussed in more detail later in this book.

CHAPTER 8

Isaac Newton: 1642-1727

Newton's Early Years.—Isaac Newton was born at Woolsthorpe, Lincolnshire, on Christmas day in 1642, the year in which Galileo died. His father, who was a small farmer, had died before Isaac was born. His early education was at the Grantham Grammar School, but he was forced to interrupt his studies when he was fifteen years old in order that he might assist in the operation of the family farm. He did not like farm work; so, later he was permitted to finish his course at Grantham. In 1661 he entered Trinity College, Cambridge, from which he obtained the master of arts degree in 1668. When asked at Cambridge why he wanted to study mathematics, his answer was,[1] "because I wish to test judicial astrology." In Newton's youth astrology was well thought of, although it soon afterward lost its standing. It was while he was at Cambridge that Newton formulated the binomial theorem. Also, in 1665, he did his first writing on fluxions or the beginning of the calculus. He also did some work on the composition of light and the theory of colors.

During 1665 he was forced to leave Cambridge because of the great plague then rampant. He returned to his home at Woolsthorpe, and it was here that he is supposed to have observed the fall of the apple and conceived the idea which later resulted in his writing of the law of universal gravitation.

In 1669, when only twenty-six years of age, Newton was made Lucasian Professor of Mathematics at Cambridge, largely because of his manuscript *De Analysi per Equation Numero Terminorum Infinatas*, which was the first exposition of his method of fluxions. This new mathematical tech-

[1] *Sir Isaac Newton and One of His Prisms*, by Rev. H. T. Inman, 1927.

nique was kept secret, except from a few pupils and friends, until the publication of his *Principia* in 1687.

Newton's Principia.—About 1672 Newton began work on his book, *Philosophiæ Naturalis Principia Mathematica,* which is commonly known as "*Principia*" and is now generally considered the most extraordinary single book in the history of the world. This book is not a recapitulation of the ideas and contributions of his predecessors but is an entirely new philosophy based on these and his own contributions to science. The formulation of these ideas and the writing of the book extended over many years. Early in its development he realized that, when and if it was published, it would start a great controversy; so, he continued to work secretly, intending that the book should not be issued until after his death. In 1684, however, Sir Christopher Wren offered Hooke and Halley a prize if they could prove that the path of a planet subjected to a force which varied as the inverse square of the distance would be an ellipse. Halley finally went to Newton with the problem. Newton immediately told him that the path of such a planet would be an ellipse, and when he was asked how he knew he replied, "I have calculated it." He was unable to find his calculations at the time but later sent Halley a practically complete discussion of the subject of motion. Halley was so impressed that he presented the document to the Royal Society.

In this accidental manner the results of approximately sixteen years of Newton's labor were discovered and published (1687) much earlier than Newton had intended them to be. Halley was responsible not only for the discovery but also for the publication, since he financed the publication out of his own personal funds. Halley, himself, considered the publication of this important work his own greatest contribution to the advancement of science. When *Principia* was published Newton was only forty-six years of age and

what later proved to be the most important work of his life was finished. His other books, on the method of fluxions (the calculus) and optics, followed.

The immediate success of *Principia* made Newton famous and established him as a great scholar. But, as is often the case, fame also brought him considerable trouble from those who tried to discredit his work.

Hooke soon claimed that he had preceded Newton with the ideas presented in *Principia;* and, when Newton published his *Fluxions* in 1693, Leibniz (1646–1716)—who had invented the Differential Calculus—also claimed priority. This latter controversy continued during the lifetime of the two principals and many years afterward among their friends.

Newton's Public Offices.—Shortly before *Principia* was published Newton began to take an interest in defending the rights of Cambridge University against the actions of King James II, and in other political activities. He was a member of the Convention Parliament during 1689–90, became warden of the mint in 1696, and in 1699 was made master of the mint, a position he held until his death. In 1701 Newton again became a member of Parliament as the representative of Cambridge University. In 1703 he was made President of the Royal Society, which position he also held until his death twenty-five years later.

During the later part of Newton's life these public offices and duties took so much of his time and energy that he had little left to devote to scientific research or study; such obligations seem to be one of the penalties great scientists and educators too often pay for their fame and success. The world is thus robbed of further scientific developments, while these able men devote their time and skill to tasks which could very often be done better by men with less scientific talent. Unfortunately, the financial rewards and

the acclaim for administrative work are usually more enticing than those accorded the man who labors long hours to discover or perfect some scientific principle.

Newton's Scientific Inheritance.—The period during which Newton lived is probably the most important epoch in the development of the Science of Mechanics. It may be well, therefore, to examine briefly the state of science and philosophy to which the developments of the earlier periods had brought the scientific men of Europe.

We have seen how the theories of Aristotle and his followers had been shaken by experimental developments and the work of Copernicus, Galileo, and others. The vague and inconsistent reasoning which they had used to explain "why" objects move and continue to move had been displaced by mathematical explanations of "how" they moved, based on concepts of time, space, and force.

Galileo had shown experimentally that once motion was established no force was required to maintain it. Force was only necessary to change either the direction or speed of the motion. Also he had shown that this motion, once started, continued because of some innate property which the body possessed but which he was unable to define. Thus, he had shown that he recognized the presence of some such properties as mass and inertia, even though he could not explain them. This development, then, was the beginning of the end of the indefinite and visionary qualities and properties assigned to matter by Aristotle; and in their place we now had the new concepts of time, space, and force, along with the new mathematical method of reasoning which Huygens had so well demonstrated by his work on the pendulum, on the center of oscillation, and on centrifugal force.

Also the relationship between science and religion was becoming considerably more friendly and cooperative. Most of the able men of science and the philosophers of the last

half of the seventeenth century were Christians who believed that science could develop without harm to religion. There were some exceptions among those who thought religion merely an accepted superstition. But the idea that the principles and concepts of science were antagonistic and opposed to religious beliefs had not yet developed.

An exceedingly important factor in the long-time growth of the intellectual environment into which Newton was born was the development of the scientific society or academy. The earliest of these was the *Accademia Secretorum Naturae* founded at Naples in 1560. Then came the *Accademia dei Lincei* (1603–1630) at Rome, of which Galileo was a member; the Philosophical or Invisible College at Gresham College, England (1645), which later (1662) became the *Royal Society of London*; and the *Académie des Sciences* of France (1666). These organizations were soon followed by many more in other countries of Europe, which helped to prepare the groundwork for a rapid growth of general scientific knowledge and a more receptive attitude toward things scientific in their respective countries. With the rapid expansion of scientific societies the development of the scientific periodical naturally followed. The first of these was the *Journal des Savants* published in Paris in 1665, and this was followed in a few months by the *Philosophical Transactions* of the Royal Society.

Such developments as these made the world into which Newton was to step much more receptive than that in which his predecessors were forced to labor. But all the dangers and difficulties were not removed from his path. The world had not yet become so accustomed to scientific development that it was willing to accept as true what seemed logical and sound to the trained scientific mind.

Marcus Marci and Galileo on Impact.—Before beginning an over-all estimate of Newton's accomplishments it may

be well to call attention to work which was performed some years before the publication of his *Principia.*

In 1639 Marcus Marci, a professor at Prague, published a treatise called *De Proportione Motus* in which the first work on impact is described. There was much other material in this book, some of which was incorrect. Some investigators seem to think that Marci obtained his ideas from Galileo. At any rate, except for the fact that his discussion of impact seems to be the first published statement on this phase of Mechanics, his book has no great importance.

Galileo had done considerable experimental work on impact, and in its course he attempted to measure the force of impact by an apparatus like that shown in Fig. 11.[2] The

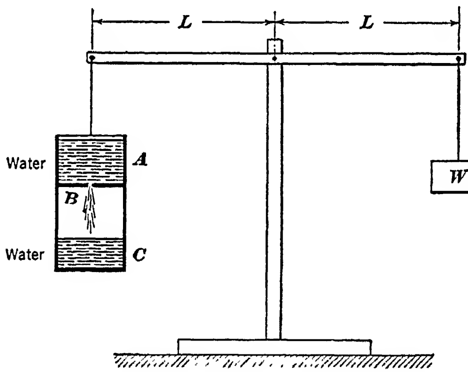

FIG. 11

weight W on the right end of the beam statically balances the water and container on the left end. With the water in compartment A, the cork is removed from the small orifice B. The water is thus permitted to flow through the orifice into compartment C.

Galileo, by means of this apparatus, expected to be able to measure the impact effect of the water as it fell into compartment C. He expected the left end of the beam to be forced down and he was going to rebalance the beam by adding

[2] *Science of Mechanics,* by Mach, p. 400.

weights to W and thus to obtain a measure of the impact effect. Instead of the left end going down there was an instantaneous downward movement of the weight W and then there was a return to static balance. This apparatus was unsuccessful, as most present-day students of Mechanics could easily have predicted. Finally after much study and speculation Galileo arrived at the conclusion that the slightest impact would overcome the greatest pressure; or, as Galileo stated it in *Discorsi*, "The force of percussion compared with the force of pressure is infinitely great."

This seems like an incorrect statement, but actually it is close to the truth. The difficulty arises from the fact that Galileo was not comparing two forces or two pressures but was comparing the quantity of motion—impulse or momentum—with a force. If we examine these quantities by means of dimensional analysis, Mv or momentum equals Mlt^{-1} and the force or pressure equals Mlt^{-2}. Thus, to obtain comparable quantities the pressure must be multiplied by t. Then we have the product of force and time, or what we now call impulse; and this quantity has the same dimensions as momentum, or Mlt^{-1}. This explanation, therefore, shows that Galileo's statement, if properly interpreted, is satisfactory; but, since it is couched in such an unusual manner, we do not immediately understand its significance. The cause of his seeming confusion of ideas was probably the apparatus he chose for his unsuccessful experiment. If he had used a simple plate and jet, he could easily have measured the impact of the jet by a pressure acting during a definite time.

Laws of Impact Discovered.—It was not until 1668 and 1669, about twenty-five years after Galileo's death, that Wallis, Wren, and Huygens, within a period of less than three months and as the result of independent investigations, stated the laws of impact in papers sent to the Royal Society in London. Wallis confined his work to inelastic bodies. Wren

checked his theory by experiments before publication. Newton referred to Wren's experiments several years later in his *Principia*, and without doubt they had considerable effect on the formulation of Newton's principles. But these principles are not basically dependent on the results shown by the experiments. Actually the only thing the experiments contribute is that they clarify the behavior of inelastic and elastic bodies. Huygens' previous investigations naturally caused him to approach the subject of impact from the "*vis viva*," or $\frac{1}{2}Mv^2$, standpoint. He found this method considerably more difficult in certain special cases and was at times obliged to approximate certain Galileo-Newton conceptions.

Importance of Newton's Principia.—The great task which Newton was to accomplish for the improvement of the Science of Mechanics may be summed up in two very brief statements:

(1) The discovery of universal gravitation.
(2) The formal statement of the principles on which the science is founded.

The thoroughness of Newton's work is demonstrated by the fact that no new basic principle has been discovered since. All the developments since Newton have been of a deductive nature and have followed paths indicated by him, and consist of further perfection and clarification of the formal mathematical statements and of improvements in the technique employed in applying the principles laid down in his *Principia*. The following quotation from Newton's preface to the first edition of *Principia* may help the reader to obtain some conception of the attitude with which he approached his great task.

"To practical mechanics all manual arts belong from which Mechanics took its name. But as the artificers do not work with perfect accuracy it comes to pass that Mechanics

is so distinguished from geometry that what is perfectly accurate is called geometrical; what is less so is called mechanical. However, errors are not in the art but in the artificers. He that works with less accuracy is an imperfect mechanic; and if any could work with perfect accuracy he would be the most perfect mechanic of all, for the description of right lines and circles upon which geometry is founded belongs to Mechanics. Geometry does not teach us to draw these lines and circles but requires them to be drawn, for it requires that the learner first be taught to describe these accurately before he enters upon geometry, then it shows how these operations, problems, may be solved. To describe right lines and circles are problems but not geometrical problems. The solution of these problems is required from Mechanics and by geometry the use of them. When so solved, it is shown and it is the glory of geometry that from those few principles brought from without, it is able to produce so many things."

From this we see that Newton's primary approach to the Science of Mechanics was geometrical. From the following further quotation from Cotes' preface to the second edition of the *Principia* we gain some additional information concerning how Newton worked.

"Those who have treated of natural philosophy may be reduced to three classes,

(1) "Of these some have attributed to the several species of things specific and occult qualities according to which the phenomena of particular bodies are supposed to proceed in some unknown manner. Aristotle belongs to this school.

(2) "Others assume that the matter is homogeneous and the variety of forms which is seen in bodies arises from some very plain and simple relations of the component particles. These often assume hypotheses as first principles of their speculations and although they afterward proceed with greatest accuracy from these

principles they may indeed form an ingenious romance.

(3) "The third class is that of experimental philosophy. These derive the cause of things from the most simple principles possible but asume nothing as a principle which is not proven. Their method is therefore synthetical and analytical. From selected phenomena they deduce by analysis the laws of nature and the simple laws of forces and then proceed by synthesis to build up the rest of the structure."

The method of Newton would place him in the third class. Newton's *Principia* in its translated form contains some 680 pages of difficult reading. The book is full of what in most cases may be called well drawn geometrical diagrams. Even an exceedingly casual and brief examination of this great book will cause the reader to marvel at the analytical ability of the author. If we are at all cognizant of the difficulties and handicaps which the pioneers in the Science of Mechanics were required to surmount, we can well wonder what would have been achieved if these great intellects had lived today with the world in its present stage of scientific development.

In *Principia* Newton starts by clarifying and reducing the work of Galileo, Huygens, and others to axioms and laws. The composition of forces by the parallelogram method Newton says is a corollary to the laws of motion. According to Newton, if a body in space is acted upon by a force, it moves in a straight line. If at the same time another force acts on it at an angle with the first force, the body takes an intermediate direction along the line of action of the resultant of the two forces. In the same general manner he develops the laws of centrifugal force and curvilinear motion, passing from a polygon to a figure with infinitely small sides.

In solving these problems Newton made full use of all the geometrical and mathematical knowledge available at the time, including that which was basic to fluxional calculus.

It will be interesting for the reader to compare the following definitions and laws as given by Newton in *Principia* with the modern versions of the same things.

(1) "The quantity of matter is the measure of the same arising from its density and bulk conjointly. Thus air of double density in double space is quadruple in quantity. It is this quantity that I mean hereafter everywhere under the name of body or mass. And the same is known by the weight of each body, for it is proportional to the weight, as I have found by experiments on pendulums, very accurately made.

(2) "Quantity of motion is the measure of the same arising from the velocity and quantity of matter conjointly. The motion of the whole is the mean of the motion of all the parts; and therefore in a body double in quantity, with equal velocity the motion is double, with twice the velocity it is quadruple.

(3) "The *vis insita*, or innate force of matter, is a power of resisting, by which every body, as much as in it lies, continues in its present state, whether it be of rest or moving uniformly forwards in a right line.

"This force is always proportional to the body whose force it is and differs nothing from the inactivity of the mass, but in our manner of conceiving it. A body, from the inert nature of matter, is not without difficulty put out of its state of rest or motion. Upon this account, this *vis insita* may, by a most significant name, be called inertia (*vis inertia*) or force of inactivity.

(4) "An impressed force is an action exerted upon a body, in order to change its state, either of rest or of uniform motion in a right line. This force consists in the action only, and remains no longer in the body when the action is over, for a body maintains every new state it acquires by its inertia only.

(5) "A centripetal force is that by which bodies are drawn or impelled or any tend towards a point as to a center. Of this sort is gravity by which bodies tend to the center of the earth.

(6) "The absolute quantity of a centripetal force is the measure of the same proportional to the efficacy of the cause that propagates it from the center through the spaces round about. Thus the magnetic force is greater in one lodestone and less in another according to their sizes and strength of intensity.

(7) "The accelerative quantity of a centripetal force is the measure of the same proportional to the velocity which it generates in a given time. Thus the force of the same lodestone is greater at a less distance and less at a greater, also the force of gravity is greater in valleys, less on the tops of exceedingly high mountains.

(8) "The motive quantity of a centripetal force is the measure of the same proportional to the motion which it generates in a given line. Thus the weight is greater in a greater body, less in a less body, and in the same body is greater near the earth and less at remote distances.

(9) "Absolute time and mathematical time of itself and from its own nature flows equably without relation to anything external and by another name is called duration; relative apparent and common time is some sensible and external measure of duration by means of which motion is measured and which is commonly used instead of true time.

(10) "Absolute space in its own nature, without relation to anything external, remains always similar and immovable. Relative space is some movable dimension or measure of the absolute space which our senses determine by the position of bodies and which is commonly taken for immovable space.

(11) "Place is a part of space which a body takes up and is, according to the space, either absolute or relative.

(12) "Absolute motion is the translation of a body from one absolute place into another, and relative motion the translation from one relative place into another."

Newton's Laws of Motion.—The three laws known as Newton's Laws of Motion are stated as follows:

(1) "Every body continues in the state of rest or uniform motion in a right line unless it is compelled to change that state by forces impressed upon it.

(2) "The change of motion is proportional to the motive force impressed and is made in the direction of the right line in which the force is impressed.

(3) "To every action there is always opposed an equal reaction, or the mutual actions of two bodies upon each other are always equal and directed to contrary parts."

Newton confirmed this third law by using two pendulums 10 feet long, as illustrated in Fig. 12. He showed that the quantity of motion, or the momentum *mv*, was constant.

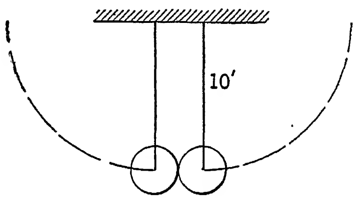

FIG. 12

Reason for Principia Undetermined.—How had Newton collected and digested the information which led to the writing of his great book? What brought it about, or what actually furnished the spark which caused him to undertake this great task, has not been definitely determined. It is known that in some manner he had discovered the idea of centrifugal force some six years before Huygens published his laws of centrifugal force. This discovery no doubt created and stimulated his interest in astronomy. Then it was natural that he should want to discover what made the planets move around the sun. The discovery of "how" they moved was a problem which the telescope had helped to solve, but there still remained unsolved the reason "why" they moved. All those who had attempted to explain the motion of the planets had tried to find the force which kept them moving.

They did not realize that no force was necessary to maintain the practically uniform motion of the planets in their orbits. Force was only necessary to change the direction of motion, as had been demonstrated by Galileo.

Probably one reason why many of the facts demonstrated by Galileo were not more generally known was because much of his original writing had been destroyed, but fortunately considerable had been saved by his students and sent to Holland where it was finally published years later.

Universal Gravitation.—It is very probable that when Newton studied the work of Galileo he obtained the idea which led to his discovery of the principle of universal gravitation, although that discovery has often been attributed to the episode of the falling apple. If the apple did fall, it was only a practical demonstration of a theory which had been growing in his mind for some time.

Newton seems to have arrived at his conception of universal gravitation through the application of the principle of continuity, which Galileo first introduced while working with the pendulum and the inclined plane. Newton studied the motion of the moon and concluded that the moon was kept from going away from the earth in a straight-line path by the same acceleration which caused bodies to fall toward the center of the earth, while its tangential velocity kept it from falling toward the earth. This analysis satisfied Newton as to why the planetary system functioned as it did. He next concluded from further application of the principle of continuity that if one of the heavenly bodies exerted accelerating forces on all the others, then the others must do likewise. This theory of universal gravitation made possible the explanation of many things which previously had been inexplicable.

The law of universal gravitation is the fundamental theory of Newton's application of dynamics to the motion of the planets. There was involved in this theory, however, a

certain property of matter which his predecessors, Galileo and Huygens, had not been able to distinguish from weight.

Concept for Mass.—Having accepted the theory of universal gravitation as correct, the next step for Newton was to obtain a concept for this undefined property which every body possessed and which influenced its motion. Newton called it *mass* and defined it as the quantity of matter which was measured by the product of the volume and density. Since density can only be defined as the mass per unit volume, this definition was not entirely satisfactory, as Professor Mach[3] demonstrated in 1883. Professor Mach says that we can only know or evaluate matter through its effects on our senses, and we can only define density as mass per unit volume. It would therefore have been better to say that if two bodies which act mutually on each other produce equal and opposite acceleration they have equal mass. A satisfactory definition of mass must be in terms of force and acceleration.

Concepts of Time, Space, and Motion.—Newton's concepts of time, space, and motion are interesting and informative, as they show the philosophical ability of the man. He recognized absolute time as something which continued or flowed on without regard to anything external, while relative or common time was some perceptible quantity of absolute time based on the relative motion of bodies and reckoned in terms of units such as hours, days, or years. To Newton time was an abstraction at which we arrived because of certain changes which took place. Absolute time was valueless to science because it could not be measured by comparison with any motion or the passage of events.

His conception of absolute space was something which always remained constant and immovable, while relative space was a changeable or movable extension or part of

[3] *Science of Mechanics*, p. 237, 264.

absolute space the conception of which resulted from our relative position with respect to other bodies or objects.

Absolute motion would then necessarily be the motion of a body from one absolute place to another absolute place, but such motion is entirely impossible. It is then evident that no person is capable of perceiving or measuring absolute space or absolute motion. They are simply concepts of thought and, since all our principles of Mechanics are based on observed or experimental knowledge resulting from the relative motions and displacements of bodies, the concepts of absolute space and motion are of no practical value to Mechanics. We derive all our ideas and concepts from the inter-relations of bodies or things one to the other. Accordingly, when a displacement or a motion takes place in any given time, we are referring this displacement or motion to the earth just as when we speak of the displacement or motion of a part of a machine (pulley, gear, or shaft) with respect to the frame of the machine or possibly to some other moving part of the machine.

Let us examine the laws which are now generally known as Newton's Laws of Motion. The relationships expressed in these laws were, as we have seen, actually discovered experimentally by Galileo. Newton merely formulated or expressed them in usable form.

The first law states that if no force acts on a body in motion the body will continue to move uniformly in a straight line. Their unawareness of this fact caused the early investigators to look for the force or forces which maintained motion. They did not know that force was needed only to change motion, not to maintain it. The planets require no application of force to maintain their practically uniform motion through space. They only need force to change their direction of motion and keep them in their paths.

The second law states that when a force acts the motion changes in speed or direction or both, and the change

is proportional to the magnitude and direction of the force. As soon as this law was formulated Newton had the answer to the "why" of the planetary system. It was simply necessary to find the deflecting or centripetal force to explain the motion of the planets.

The third law states that when a body exerts a force on another body the other body exerts an equal and opposite force on the first body, or there is action and reaction. Much confusion of thought resulted from the incorrect interpretation of this law. An illustration of this type of thinking is furnished by the oft-repeated example of a horse pulling a cart. They say that when a horse pulls a cart the cart pulls back on the horse with an equal force. Why then does the cart move? The cart moves because the horse exerts an unbalanced force on the cart. The statement is then made that the cart is pulling back on the horse for, if it did not, what would the horse be doing? The apparent confusion results from the fact that the forces are acting on two different objects (two free bodies). If both the pull of the horse and the pull of the cart acted on one body (the cart or the horse), then that body would be in equilibrium and no motion would result. This third law is also a statement of the principle of conservation of energy when it is applied to two bodies which do not move.

Gravity and the Moon.—The second law of motion is the most important of the three, as it is really the fundamental law of dynamics. With these basic laws formulated, Newton soon realized that the explanation of the functioning of the planetary system was forces coming from the sun and varying inversely as the squares of the distances to the planets from the sun. Such a force would cause circular motion, but he also knew that the planetary motion was elliptical, not circular. He proceeded to attack the problem with his fluxions and Cartesian geometry and was able to demonstrate that,

if the sun was at one of the foci of the ellipse, then the planet would travel in an elliptical path when the attracting force varied according to the inverse-square law. The problem now was to find this attracting force. The falling apple is supposed to have suggested gravity as the force. Newton immediately (1666) attempted by calculations on the moon to prove that gravity was the force which caused the moon to follow its elliptical path.

Since, according to his assumption, gravity (the attracting force) varied as the inverse square of the distance, and since the moon was 60 earth's radii distant from the earth, then the force of gravity at the moon would have $\frac{1}{3{,}600}$ times its magnitude at the earth's surface. So, instead of the moon falling $s = \frac{1}{2} \times 32.2(1)^2 = 16.1$ feet in one second, as it would at the earth's surface, it should fall $s = \frac{\frac{1}{2} \times 32.2(1)^2}{3{,}600}$ feet per second, or $s = \frac{\frac{1}{2} \times 32.2(60)^2}{3{,}600}$ or 16.1 feet per minute. However, when Newton attempted to check this figure, using 60 miles as the length of a degree, he found that he obtained only 13.9 feet per minute instead of 16.1 feet per minute. He was unable to explain the discrepancy; so, according to his own words, he "laid aside at the time any further thought of the matter." He was only twenty-three years of age then, and so he still had plenty of time for the solution of such problems.

During the year of his election to the Royal Society (1672) a paper by Picard of Paris on the determination of the size of the earth was presented at one of the meetings. How long it was before the contents of this paper reached Newton is not known, but when they did he immediately discovered the reason for the error in his early calculations. In this paper Picard established the length of a degree as slightly less than 70 miles, while Newton had used 60 miles. Newton now repeated his earlier calculations and found that with the

new distance the results were satisfactory. He was convinced that his theory of the mechanism of the universe was sound. He immediately went to work on the problem in all its details. He worked so intensely and continuously at this task for the next two years that he became mentally ill and was forced to rest. However, by this time the work had progressed to such an extent that others were able to carry on with the task.

There seems to be some disagreement about the error which Newton discovered and also why he did not proceed with his work when he first started it. Professor Cajori says that there were several good estimates of the size of the earth available to Newton. One by Gunther, whose book Newton was known to have purchased, gives the size of a degree as 66⅔ English statute miles. If Newton had used Gunther's value, he would have had an error of only 3½ per cent, which might have given rise to his statement that "he found the answer pretty nearly." It has been suggested by some that the reason Newton laid aside his work in 1666 was because of the great difficulty of making the calculations.

Newton's solution was attacked by Huygens and Leibniz because he offered no explanation for the cause of the gravitational force. No doubt he was anxious to place his discovery in the records before someone else arrived at the same conclusion. He had found something that worked and which could be expressed mathematically. The cause of the gravitational force could wait for future developments and would be explained when all phases of the theory had been worked out.

Newton's Contributions to Mechanics.—In order that the reader may know Newton's religious feeling at this period of his life, the following quotation from the second edition of *Principia* is given: "This most beautiful System of the Sun, Planets and Comets could only proceed from the counsel and dominion of an intelligent and powerful Being—God endures forever and is everywhere present and by existing always

and everywhere he constitutes duration and space." It might also be taken as his definition of time and space.

In the period between 1666 and 1672 Newton devoted much of his time to optics. His work in this field brought out so many controveries that, when he started writing *Principia* (about 1672), he decided he would not have it published until after his death. But, as has been related, circumstances and Halley altered this plan.

The accomplishments of Newton in the field of Mechanics are summed up in his definitions and laws. The most important of these are the concept of mass and the principle of universal gravitation. This principle explains the mutual attraction of bodies which causes acceleration and is dependent on space and the material constituting the bodies. Professor Mach[4] sums up the whole thing by saying that in reality only one great fact was established: "Different pairs of bodies determine, independently of each other, and mutually, in themselves, pairs of accelerations, whose terms exhibit a constant ratio, the criterion and characteristic of each pair."

Newton's principles were not expressed in algebraic terms because up to this time all such matters had been described in geometrical language, vectors being used for forces. If he had used algebraic symbols, he probably would have written $F=Ma$ as $Ft=Mv$ or $F=\frac{Mv}{t}$. This fact may explain why Newton gives no formal statement of the principle of work and energy, since $a=\frac{v^2}{2s}$ cannot be shown geometrically. That is, when $F=Ma$ and $a=\frac{v^2}{2s}$ are combined, we obtain $Fs=\frac{1}{2}Mv^2$, or the work and energy relationship which was the method used by Descartes, Huygens, and Leibniz to express the effect of a force.

[4] *Science of Mechanics*, p. 307.

Soon after Galileo had finished his investigations of falling bodies, it became apparent to some that a moving body possessed the ability to overcome a resisting force or that it could accomplish that which a force could do, namely, resist or overcome a force. Naturally, then, the question arose as to how this effect, or—what was apparently the same thing—the effect of a force, was to be measured or determined.

These two ways of measuring the effect of a force, namely, $F = Ma$ and $Fs = \frac{1}{2}Mv^2$, brought about another bitter controversy between the Galileo-Newton followers and the Huygens-Leibniz followers. The Galileo-Newton group maintained that $Ft = Mv$ was the correct measure of the effect of a force, and the Huygens-Leibniz group contended that it was $Fs = \frac{1}{2}Mv^2$.

The English used Newton's geometrical presentation and the French, Germans, and Swiss followed Huygens, using Leibniz' calculus. This controversy went on until D'Alembert published his *Traité de Dynamique* in 1743, showing that both were correct but did not mean the same thing.

Newton demonstrated for all time that the function of a force was to cause acceleration or to change the shape of a body. In order to study the flux or changing relations of force it was necessary that he invent his method of fluxions which later developed through contributions by Leibniz and others into our present-day calculus. His first formal presentation of his early ideas on fluxions he made to his students in 1669 as *De Analysi per Equation Numero Terminorum Infinatas.*

Before Newton's time the Science of Mechanics consisted of the theory of statics as developed by Archimedes and Stevinus and the theories of dynamics as developed by Galileo and Huygens. Newton correlated these theories, formulating the three laws of motion and thereby founding the system of Mechanics which has been sufficient to explain the mechanical phenomena of the universe and has withstood the use and refinement of three centuries.

The many principles which had been evolved during the development of the science were largely results of the fact that the different investigators had reached their conclusions by degrees. In reality the only fact established was that $F=Ma$. Not even the super-minds of Galileo, Huygens, and Newton were able to perceive this simple relationship in its entirety. They discovered it part by part in the laws of falling bodies, the law of inertia, the parallelogram of forces, the concept of mass, etc.

The form of the equation $F=Mg$ is due to its historical background, the order in which the contributing parts were discovered. If this relationship had been formulated at a later date, it might have turned out as $Fs=\frac{1}{2}Mv^2$ or the relationship of work and energy; but, because Galileo had discovered $v=gt$ first, the form $F=Ma$ or $F=Mg$ was the natural result.

First came $v=gt$, which was followed a bit later by $s=\frac{1}{2}gt^2$ and finally by $s=\frac{v^2}{2g}$. If each of these three equations in turn is combined with $F=Mg$, we obtáin the relationships

$$Ft=Mv$$

$$Ms=\frac{Ft^2}{2}$$

$$Fs=\frac{Mv^2}{2}$$

When examined, these relationships lead to the following conclusions:

If the concept of force is the basic factor and time is the factor which determines velocity, then work becomes a derived quantity.

If work is the basic concept and distance is the factor which determines velocity, then force becomes the derived quantity.

Newton and Galileo preferred the first method and Huygens the second. Newton works exclusively with force, mass, and momentum. His appreciation of the concept of mass shows that he was a bit more competent in his thinking than either Galileo or Huygens, who made no distinction between mass and weight.

The two methods, $F = Ma$ and $Fs = \frac{1}{2}Mv^2$, could have been developed independently, but it did not happen that way. They were more or less merged along the way because of the access of the investigators to each others' ideas. Newton was all sufficient with his force, mass, and momenta; and Huygens could have carried on with his work and mass, but he unfortunately did not understand mass.

What, then, should Newton be credited with, in addition to his over-all synthesis of the Science of Mechanics as it appears in his *Principia*? Probably most students of the development of the science would be willing to concede that Newton was either the contributor or a collaborator in the establishment of the following concepts and laws:

(1) A clear concept for force
(2) Concept for mass
(3) The theory of centrifugal force or of curvilinear motion
(4) Theory of the attraction of masses
(5) Laws of motion
(6) Theory or method of fluxions
(7) Law of universal gravitation
(8) General formulation of the law of the parallelogram of forces.

Such a contribution to the basic theory of any science could only be made by one of the world's greatest minds.

Thus, looking back after these many years at the period of Newton's lifetime, we find that it was not only the period during which the first satisfactory synthesis of scientific

knowledge was made but was also a period during which the viewpoint of man intellectually had to be revolutionized, especially the dogmatic code of religious belief, if both religion and scientific progress were to carry on and each fulfil its necessary and important part in the rounded growth of human life. This was brought about not so much by the work done in Mechanics as by the developments in other fields of science.

Before leaving Newton, let us see what he, himself, thought of his accomplishments when he was approaching the end of his time on earth and had received many honors. "I know not what the world will think of my labours, but to myself it seems that I have been but as a child playing on the seashore: now finding some pebble rather more polished and now some shell more agreeably variegated than another, while the immense ocean of truth extended itself unexplained before me." This is indeed a modest estimate of his contributions to the storehouse of scientific knowledge, but at the same time is a provocative challenge to those who were soon to be left with the responsibility for the further development of the science which he had coordinated.

CHAPTER 9

Work of Newton's Contemporaries and Followers

In the preceding chapter, we have attempted to give the reader, in a rather limited space, an accounting of the accomplishments in a particular field of one of the world's greatest thinkers. After all these years many of the details are missing, but to the reader of the present day these are not essential. However, it is this writer's opinion that Newton was not the king or knight in the gilded armor who led the warriors of science during this particular period of the world's scientific development. He undoubtedly was the crown prince or the relief man who kept the battle going in the right direction long after the real leader had become a casualty, not only of the cause but of the passage of years. It is my belief that it was Galileo's accomplishments and inspiring leadership which carried the fight forward long after he had ceased to be among those present.

As has been previously mentioned, whenever in the history of man an outstanding thinker has raised his head above the crowd, inevitably a group of lesser satellites surrounds or follows close after him.

There are several outstanding contributors to the development of Mechanics who either were contemporaries of Newton or came some years later, and whose work we shall now examine.

Principle of Moments.—In 1687, the year in which Newton's *Principia* was published, Pierre Varignon (1654–1722) also completed a manuscript for a book called *Project d'une Nouvelle Mécanique.* This was a new type of statics which was built up from what has now become known as

the principle of moments. This work was not published in book form until after Varignon's death.

Varignon starts out by presenting the definitions, hypotheses, and axioms upon which he later builds his system of Mechanics. He assumes that all parts of the mechanisms which he investigates are weightless, perfectly rigid or undeformable, and frictionless. He works only with the externally applied forces and disregards entirely the effect of the mass of the parts of the machine. With these assumptions established, he then proceeds to develop a logical system of Mechanics.

Varignon defines Mechanics in the following manner: "Mechanics is in general the science of motion, of its cause, and its effects; in a word, of all that pertains to motion. Consequently, it is also the science of machines."

In the development of his system he arranges the material in his book in the following order:

(1) Axioms, hypotheses or postulates, and propositions
(2) Weights supported by cords
(3) Pulleys
(4) Wheel and axle
(5) Levers
(6) Inclined plane
(7) Screw
(8) Wedge
(9) General principles of the simple machine
(10) Equilibrium of fluids

Varignon demonstrates the principle of statical moments by geometrical diagrams. He represents his forces by vectors and then shows that the product of a force and the perpendicular distance from some point (lever arm) is represented by an area. He continues his development of the principle of moments by showing the equality of the area which represents the moment of a resultant force and the algebraic sum of the areas representing the moments of the components of

the resultant force. In this manner he established the truth of the principle which has since become known by his name —Varignon's Principle of Moments. Throughout his book he refers all problems in equilibrium back to this demonstration of the theorem of moments.

Varignon also was an able mathematician outside the field of geometry. In 1700[1] he evolved the differential equations

$$\frac{dx}{dt}=v \qquad dv=a\,dt$$

$$\frac{d^2x}{dt^2}=a \qquad v\,dv=a\,dx$$

Also he was the first to express the velocity of flow[2] of a fluid as $v=(2gh)^{\frac{1}{2}}$.

His contributions to the Science of Mechanics can be summed up as follows:

(1) Proof of the principle of moments

(2) A system of statics founded on moments

(3) The differential equations $\frac{dx}{dt}=v$; $\frac{dv}{dt}=a$; $\frac{d^2x}{dt^2}=a$; $v\,dv=a\,dx$

(4) $v=(2gh)^{\frac{1}{2}}$ for velocity of flow.

Uniform Notation and Beginning of Cooperative Research.—Previous to 1700 the transmitting of scientific knowledge from one country to another was a difficult matter, both because of distance and because of the difficulties of making the new knowledge readable in different languages.

After the principles of statics and dynamics as developed by Stevinus, Galileo, Huygens, Newton, and Varignon had become widely known, there began to develop an interchange of ideas among the leading workers in Mechanics in different countries. This might be called the beginning of cooperative research. It soon brought about the realization that the transmission of ideas would be greatly facilitated by a uni-

[1] *Memoirs de l'Academie des Sciences de Paris*, 1700, p. 22.

[2] *Science of Mechanics*, by Mach, p. 494.

form notation which could be easily understood in all the countries of Europe. This is one of the more important pioneering steps which have resulted in the development and perfection of our present analytical and mathematical methods. This growth started from the crude diagrams of the early workers, such as Stevinus, and developed slowly down through the years. The process was now greatly accelerated by the offering of prizes by the Philosophical Societies of Berlin, Paris, London, and St. Petersburg for the solution of special problems.

The exchange of problems not only developed the Science of Mechanics (incidentally the calculus was much improved as a mathematical tool) but it produced what might be considered an even greater long-time effect on science in general. It was probably the beginning of the idea which has developed into our present-day engineering and scientific societies and international conferences which have done so much to stimulate the growth of scientific knowledge.

These exchanges helped to develop and clear up certain fundamental ideas and concepts, such as what constituted mass, and they also brought to the attention of the workers new subjects which required the attention of many minds before agreement could be reached.

Equation of the Catenary; Virtual Displacement.—The Bernoulli brothers, James and John, were prominent figures in mathematics during this period. James (1654–1705) was the first to develop the equation for the catenary. After solving the catenary or chain problem he presented it as a challenge problem in 1691 and obtained correct solutions from three others, his brother John, Leibniz, and Huygens. These solutions were published by the Royal Society at London in 1697. When Bernoulli attempted to solve the problem he realized that equilibrium would be established when the various links had adjusted themselves to such

positions that none of them could attain any lower position. The problem then became simply one of determining the equation of the curve which had the lowest center of gravity for a given horizontal span. This equation he was able to obtain by calculus. He also used the calculus method to determine the equation of curvature for a cantilever beam with a concentrated load at the free end. This was probably the beginning of the double-integration method for determining beam deflections. Many such problems were presented and solved by various members of the group of able mathematicians of this particular period. The solutions of the problems greatly stimulated the development of the calculus as a working tool of Mechanics and science in general.

John Bernoulli (1667–1748), in addition to being the leading mathematician of his time, contributed articles on many other subjects of scientific knowledge. His principle contributions to Mechanics were the introduction of the symbol g for gravity and the principle of virtual velocities which he presented to Varignon in a letter in 1717. This idea of Bernoulli's really depended on the principle of virtual (possible) displacements which had first been observed by Stevinus near the end of the sixteenth century while he was making investigations of the conditions required for the establishment of equilibrium of various combinations of pulleys.

In his letter to Varignon, John Bernoulli stated that when equilibrium exists in any system of forces subject to restraints, such as when a system of forces acts at various points on a linkage,

$$P_1p_1+P_2p_2+P_3p_3=0 \quad \text{or} \quad \Sigma Pp=0$$

In these equations P, P_1, P_2, and P_3 are the forces and p, p_1, p_2, and p_3 are the displacements of the points of application of the forces in the directions of the forces.

The term virtual displacement was intended to mean the displacement in the direction of the force without disturbing the equilibrium of the system. These displacements in the direction of the force were called virtual velocities by Bernoulli.

The confusion in terms was probably caused by translation from the Latin to other languages. Actually the principle is nothing but an extension of the old rule that what a machine gains in power it loses in velocity or, in terms of work, the product of the force and the distance is a constant.

John Bernoulli seems to have been the first to use the term "integral" in his calculations, and he also was the first to attempt to formulate a set of rules for use in integral calculus instead of working out each problem of integration independently.

Mathematical ability of note continued for three generations in the Bernoulli family. The two brothers, James and John; three sons of John, namely, Nicholas, Daniel, and John Jr.; and two sons of John Jr., namely, John 3rd and Jacob, all attained distinction as mathematicians. Of the younger generation Daniel (1700–1782) was the most prominent, winning the French Academy prize ten times. His work in Mechanics was principally in hydrodynamics.

Moment of Inertia; Analytical Mechanics.—Leonhard Euler (1707–1783), a Swiss mathematician who was a friend of John Bernoulli and his family, did much to improve integral calculus. His ability was chiefly in pure mathematics, although he did make a number of contributions to Mechanics. The better known of these are in that branch of Mechanics now known as Strength of Materials. While Professor of Mathematics at St. Petersburg, he published in 1736 what has since been generally granted the distinction of being the first book on Analytical Mechanics (*Mechanica sive Motus Scientia Analytice Exposita*). In this book he

used much of the old geometrical procedure but also introduced the method of resolution of forces into three components X, Y, and Z. Euler was also the first to use the name moment of inertia for the term $\int r^2\, dm$. Huygens had used the expression but had not considered it important enough to give it a name. Euler had also developed the relationship now known as the moment-of-inertia transfer formula. He worked out the expressions for the moments of inertia of a number of the common bodies such as the sphere, cone, and cylinder, and also for their radii of gyration, and is credited with being the first to use the Greek letter π (pi) for the ratio 3.1416.

Euler did some work on least action or maximum and minimum during equilibrium, and stated the principle of least action in a rather indefinite manner. He is also believed to have contributed to the development of the principle of *vis viva*, which states that in any motion the change of *vis viva* is independent of the form of the path (frictionless path).

The D'Alembert Principle.—It was fifty-six years after the publication of Newton's *Principia* (1687) that Jean le Rond D'Alembert (1717–1783) published his *Traité de Dynamique* (1743). Much had been accomplished during that period through the solution of special or particular problems and in the perfection of methods and technique; many of the developments must be credited to the controversies started and the exchange of problems between the brilliant mathematical minds of the period.

D'Alembert realized that all this knowledge must be brought together, evaluated, and arranged, if it was to be of service to the scientific workers of the world. That is what he proceeded to undertake in his *Traité de Dynamique*. However, this book is not merely a gathering together of facts and principles developed by others; it also presents an entirely new and original idea, that of solving problems in

dynamics by applying the principles of statics. This is accomplished through the development and application of what is now known as the D'Alembert Principle. This principle has been stated in several different ways in the years which have passed since D'Alembert first presented it. In his *Traité de Dynamique*, D'Alembert states it in the following manner:

If on any system of points M_1, M_2, and M_3, Fig. 13, connected together by links or constraints, the forces P_1, P_2, and P_3 are impressed, these forces would give the points

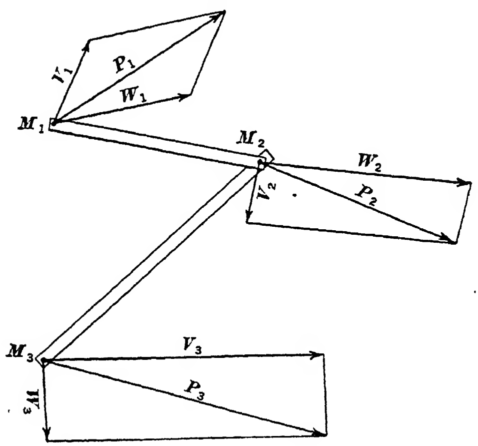

FIG. 13

M_1, M_2, and M_3 certain definite motions if the points were free and unrestrained; but, because of the restraints the points M_1, M_2, and M_3 attain motions which could be produced by forces W_1, W_2, and W_3. Now, if each of the forces P_1, P_2, and P_3 is resolved into components W_1, W_2, and W_3 and V_1, V_2, and V_3, then only the components W_1, W_2, and W_3 are effective, and the components V_1, V_2, and V_3 are equilibrated by the connections or restraints. The forces P_1, P_2, and P_3 are called impressed forces, and the forces W_1, W_2, and W_3, which produce the actual motion, are the effective forces.

Since the equilibrated forces form a system balanced by the restraints, only the effective forces remain unbalanced. This is the D'Alembert Principle as stated in *Traité de Dynamique*, except that he uses momenta instead of forces.

The general meaning of this principle is that the effective forces, if reversed (inertia forces), are in equilibrium with the applied or impressed forces.

With the publication of *Traité de Dynamique* and the demonstrated application of the D'Alembert Principle to the solution of problems of dynamics, Mechanics had come of age and was demanding recognition as an independent science. It had now developed to the point where its power and prestige required that it be considered as a primary branch of science and not just a phase or part of mathematics or physics. D'Alembert cannot be given sole credit for the development of the principle which bears his name, as James Bernoulli in *Acta Eruditorum* (1691) had expressed some ideas on the subject. Also the principle is a by-product of Newton's second law. But D'Alembert first put it into formal shape and announced it as a workable principle.

In addition to presenting this principle, D'Alembert's *Traité de Dynamique* was important because it put Newton's work into the calculus form and opened the way for a complete and much needed analytical treatment of the Science of Mechanics. In fact this was to be the next forward step in the science, Newton's geometrical treatment being entirely too cumbersome to satisfy the practical worker.

Plane Friction.—Charles Augustin Coulomb (1736–1806), scion of a long line of magistrates considered country nobility, was born June 14, 1736, at Angouléme, France. He was educated in Paris and then became an officer in the engineers corps. He spent three years at Martinique in the West Indies and then returned to Paris because of ill health. In Paris he became interested in science.

Coulomb was a natural experimental genius who was always trying to carry his investigations a bit further than anyone before had gone. He soon built up a reputation as a scientist of exceptional ability. In 1779 he won an Academy award for a paper entitled *Theorie des Machines Simples*, in which he published for the first time the laws of sliding friction and also those of torsion as applied to a torsional pendulum. Coulomb's friction experiments were performed with weights of several hundred pounds because he desired the data for military purposes. Coulomb's interests were not confined to any one phase of science. He was especially able in electricity and Mechanics of Materials. His important work in Mechanics of Materials will be discussed later.

Analytical Mechanics Improved; Least Action.—Joseph Louis Lagrange (1736–1813) was born in Turin, Italy, of French parents. He became a professor at the Artillery School at Turin, and later at the Polytechnic School of France, and finally became Director of the Berlin Academy. He also was a Senator of France and Count of the Grand Cross of the Legion of Honor. He possessed great mathematical ability, especially in the field of pure mathematics. Five years after D'Alembert's death Lagrange published his *Mécanique Analytique* (1788). This was the first entirely analytical treatment of the subject as opposed to Newton's geometrical method (*Principia*). There are no diagrams in this book; it is entirely mathematical. Thus, we see that a hundred years were necessary to produce sufficient development of the calculus, the workable notation and mathematical technique to transfer Newton's laws from geometrical diagrams to mathematical equations.

The introduction to Lagrange's *Mécanique Analytique* shows how confident he was that this new type of Mechanics would be received with general satisfaction and acclaim. The following quotation is from that introduction:

"This work will have in addition another advantage; it will collect and present under the same point of view the different principles, so far found to facilitate the solution of mechanical questions. It will show their connection and their mutual dependence, leaving one to judge of their accuracy and value.

"No diagram will be found in the work. The method which I follow requires neither figures nor arguments geometrical or mechanical, but merely algebraic operations arranged in a regular and uniform order. Those who are fond of analysis will anticipate this Mechanics with pleasure and be pleased that I have set it forth in this way."

Lagrange also must be given credit for developing the form of solution known today as the method of work and energy, although Euler (1744) probably was the first to use it when he made his contribution to the solution for the change of the *vis viva* on any path; or it might have been Huygens during his investigations of the pendulum. However, neither Huygens nor Euler developed the relationship into a general principle. Also the term "work" was first used by Coriolis in 1829.

It was Lagrange who demonstrated that here was another general theorem which could handle any problem involving moving bodies just as effectively as Newton's $F=Ma$. Lagrange's statement of the theorem was not in the form commonly used in our Mechanics books of today. His forces were expressed in terms of their components parallel to the three coordinate axes. The theorem therefore appears to be considerably more complicated. In writing about the principle he makes the following statement:

"Such is the principle to which I give here, although improperly, the name 'least action,' and which I consider not as a metaphysical principle but as a simple and general result of the laws of Mechanics."

Couples; Conservation of Energy.—Lewis Poinsot (1777–1859) was born in Paris and attended the École Polytechnique. After graduation he became an engineer of roads and bridges, and in 1804 was made professor of mathematics at Lyceum Bonaparte. He returned to École Polytechnique as Professor of Mathematics in 1809 and was elected to the Institute in 1813.

Poinsot's contribution to the Science of Mechanics was his theory of couples which he first presented in his *Éléments de Statique* published in 1803 and again in *Nouvelle Theorie de la Rotation des Corps* published in 1834.

In *Éléments de Statique* he gave the usual demonstration now used to show that a force is equivalent to a single force and a couple and that the couple can be moved about in its plane or in any parallel plane; he also showed that any system of forces in space may be compounded into a resultant force and a resultant couple.

This work of Poinsot presents the development of the final fundamental principle of Mechanics as it is known today, except for the principle of conservation of energy and matter which cannot be credited to any one individual, but is rather the result of the accumulated experience of many workers and of observations and experiments extending over a great many years. Even Descartes (1596–1650) and Newton (1642–1727) seemed to have some rather vague ideas on conservation. Newton in the discussion of his third law says: "If the action of an agent be measured by the product of the force into its velocity and if similarly the reaction of the resistance be measured by the velocities of its several parts multiplied into their forces, whether they arise from friction, adhesion, weight, or acceleration, action and reaction in all combinations of machines will be equal and opposite."

The complete and final proof of the law of conservation of energy was so closely connected with the relationship of mechanical energy and heat that its final confirmation

depended on the development of satisfactory and accurate thermal measurements. As our knowledge of thermodynamics increased and the experimental technique improved, experimental results approached closer and closer to that which scientists agreed was true but which they had not been able to show experimentally.

By 1860 the thermal equivalent of mechanical work had been determined accurately enough to satisfy science of the truth of the principle of conservation of energy. The value of this thermal equivalent of mechanical work was still to be modified as scientific technique progressed.

We cannot here go into the momentous significance of the basic law of conservation; its importance to physics, chemistry, and thermodynamics is too well known to all workers in these fields to need comment. It seems, however, almost beyond human conception when we realize that thousands and thousands of years of human development plus about 2000 years of geometrical Mechanics and finally approximately 100 years of analytical or mathematical Mechanics had to be behind man before he was able definitely to formulate this principle of the conservation of energy. Now that we have this great tool, how much further will man go and how rapid will be his future progress? On the basis of the accomplishments of the last fifty years, we should be able to visualize a rather optimistic picture of what still may come.

CHAPTER 10

In Retrospect

We have traced the progress of the Science of Mechanics from its beginning of crude tools and implements in the hands of illiterate, untutored men to the present day, a span of about 6000 years. We may think of this as a long time, and that progress has been indeed slow. Actually, however, 6000 years is but a day in the life span of the world. Further, when we realize that Mechanics, in the form in which we engineers of today employ it, is only about 200 years old, we can well regard it as little more than an hour in the day of the slow development of the human race.

The purpose of the author has been not to enter into technical detail in examining the creation of the science we know now as Mechanics, but to try to give the student or engineer a better idea of the struggles and accomplishments of the pioneers in this field, and to trace the development of the more important concepts and principles which have brought into being the Science of Mechanics.

Someone has said that science is human experience tested, arranged, and coordinated into usable laws and formulas. We started with the crude hand-made implements whose creation was forced upon man by the sheer necesstiy of self-preservation, and then we passed on to the queer but interesting attempts of the early Greek philosophers such as Aristotle to explain the movement and behavior of bodies when acted upon by forces—their strange concepts of why objects move with "natural" or "violent" motion, or their theories that objects "float because of their shape" and that "heavy bodies go down because they belong down." These theories now seem absurd to us, and we wonder that men with such brilliant minds as were possessed by Aristotle and

his associates could offer such basically false solutions to problems for which almost every college student of today knows a better answer. But when we remember that our present-day knowledge is built upon a firm foundation of sound concepts (which the early Greeks did not have) and rational thinking checked by actual experimentally obtained results, then we begin to realize how difficult the tasks of the founders of our basic sciences were, and we are not surprised that their theories were often in error.

Even the work of Archimedes from a pure scientific viewpoint was not of much importance in the development of Mechanics. His most valuable contribution was probably the creation of a science of measurement and its application to the study of ordinary natural phenomena. When experimenters were able to measure, then they were able to compare and make estimates. Thus the Science of Mechanics was born.

Development of Basic Concepts.—The next step was the concept of force. At first force was simply thought of as the result of the muscular vigor of some unseen demigod. Later other concepts of force were established, and finally force lost its connection with the supernatural and became associated with mass. Then followed the concepts of time and space. They are fundamental and cannot be compared with anything else. Therefore, they cannot be defined in terms of any other quantities. The discussions of absolute time and space belong to metaphysics, and the natural philosophers and their concepts are of no practical value to the engineer. Much has been said and written about these concepts, but we as engineers have no occasion to deal with them. Engineers need to know only about relative time and space.

Along with the fundamental concepts previously mentioned, we must have one other—that of matter or mass. The fundamental concept of matter had passed through

many hands and had appeared in many books under different names, such as "corpus" and "moles," before a distinction between weight and mass was finally arrived at.

Clerk Maxwell[1] (1831–1879) said: "All we know about matter relates to the series of phenomena in which energy is transferred from one portion of matter to another till in some part of the series our bodies are affected, and we are conscious of a sensation. We are acquainted with matter only as that which may have energy communicated to it from other matter." According to this statement matter becomes a medium for the transmission of force.

In 1867 Thompson and Tait[2] made the following statement about matter: "We cannot, of course, give a definition of matter which will satisfy the metaphysician, but the naturalist may be content to know matter as that which can be perceived by the senses or as that which can be acted upon or exert a force." Here again we find the concept that matter is simply a medium for the transmission of force or energy, and this concept is satisfactory for the purposes of the Science of Mechanics. Tait[3] also tells us that we shall probably never know what matter is and that it is beyond the range of human intelligence to define it.

As we look back over the years and examine what remains of the work of the great pioneers of the Science of Mechanics, we see that the development of science during this early period was very much an unguided, haphazard process. It was not carried on by well organized and equipped staffs of trained workers such as the large modern research laboratories of today have at their disposal. Progress was made as a result of the zeal, courage, and brilliant mental capacity of individuals who carried on their search for knowledge, even though they were usually handicapped badly by the many

[1] *Matter and Motion,* p. 163.
[2] *Natural Philosophy,* p. 207.
[3] *Properties of Matter,* p. 12–13 and p. 287–291.

material and mental hazards thrown in their path by various religious and political groups.

An interesting commentary on the influence of religion on present-day scientific thought is made by Harold J. Laski, Professor in the London School of Economics and Political Science, in his chapter on the Spirit of Our Age in the book *Revolution of our Time.*

"The decay of the religious spirit is widespread. But if we seek for a religious revival, we must be careful to define our terms with some precision. If, thereby, we mean a revival of faith in the supernatural, the evidence is clear that, especially in any of the historically dogmatic forms, it is unlikely; for their power to offer rational proof of their title to acceptance dwindles with every advance in the scientific understanding of the universe. There is, moreover, no form of cruelty in human experience to which the religious spirit has not been able to accommodate itself; there is a grim truth in the accusation that its votaries have been content, for the most part, to see it operate as the opium of the people. There is, again, no great advance in human knowledge of which the classic religions have not been the uncompromising opponents until its truth was so obvious that some shame-faced accommodation had to be sought."

The development of those concepts which we now recognize as the basic laws and principles of the science was a slow process of evolution which extended over many years. These laws and principles were formulated as such only after many of the facts involved had long been known.

Experimentation Takes the Torch.—The entire structure of our modern science which has grown up since the Renaissance has been based on the union of intellectual activity and manual work.

Aristotle, a pupil of Plato, was the greatest biologist, logician, and systematizer of knowledge of the ancient times.

Yet he rejected the atomic theory and believed that the essence of matter was to be found in its qualities which—according to him—were hotness, coldness, wetness, and dryness, and he also believed that these qualities when united in binary combination would produce water, earth, air, and fire.

Thomas Aquinas' work was filled with Aristotle's logical deductive type of knowledge because the Schoolmen thought that by following such a procedure they had built a philosophy on the impregnable base of the Holy Scripture, the Fathers of the Church, and the rediscovered works of Aristotle. The few medieval experimentalists like Roger Bacon with their isolated facts were ineffective against such complete rational philosophy.

Even Aristotle had observed that when two weights balance each other, if they move at all, they will move at speeds that are in inverse proportion to their weights. He also was aware that there was some factor which changed the effect of forces when applied to different bodies (what we moderns call mass). He knew it was there or existed, but what it was he did not know. His lack of comprehension of the concept of mass brought forth such statements as the following[4]: "Why is it that neither very small nor very large bodies go far when we throw them? Is it that what is thrown or pushed must react against that which pushes it and that a body so large as not to yield at all or so small as to yield entirely and not react produces no push?" Others made statements such as the following[5]: "If the force of a body depends on its velocity as it appears to do, how is it that a body at rest has any force at all and how can it resist the slightest effort or exert any pressure?" and "If one man can draw half of a certain weight and another man also one-half, when the two act together these proportions should be

[4] *History of Inductive Sciences*, by Whewell, p. 334.
[5] *History of Inductive Sciences*, by Whewell, p. 335.

compounded so that they ought to be able to draw one-half of one-half or one-quarter only." Even as late as 1639 a certain investigator by the name of Arriga was so badly confused that he was unable to explain why several weights placed one on top of the other on a horizontal plane were able to produce a greater pressure against the plane than could be produced by the lowest weight acting alone. Another fallacious theory, presented by Aristotle, was that a body ten times the weight of another would fall ten times as fast. It is now rather generally conceded that much of this confusion might have been avoided if the early Greeks had confined their attention to bodies at rest and attempted to master the theories of statics before they sought to explain the mysteries of dynamics. The fact that they failed to realize that the Science of Mechanics inherently consists of two parts, statics and dynamics, brought forth much unsound reasoning which naturally led to a great deal of confusion.

It was Galileo, with this leaning tower of Pisa experiments (1590), who initiated the scientific skepticism which finally brought about the overthrow of the erroneous theories of Aristotle, and started the crusade for scientific thought based on facts experimentally demonstrated as true, in opposition to the former policy of theories based on speculation alone with no practical proof of their validity. Others, such as Descartes and Francis Bacon, helped to carry the torch for this new regime, but Galileo was the first crusader for the new order, which has so successfully carried on down to the present day and will no doubt continue into the future.

With the establishment by Stevinus of the laws of the inclined plane, the laws of equilibrium for such simple machines as the wedge and the screw easily followed. The principle of the pulley was deduced from that of the lever. With this work done the basic problem of equilibrium was solved and the investigators were better prepared to attack dynamics.

While working with pulleys, Stevinus had observed that the products of the weights and their displacements were equal. Galileo had also observed a somewhat similar condition in connection with his work on the inclined plane. Both of these incidents were demonstrations of what was later called the principle of virtual displacement. Also, in connection with the lever, Galileo had established the fact that in raising a weight by a lever or other machine, what is gained in speed is lost in force. This was a statement of the principle of virtual velocity. Galileo, however, was not the first to observe this fact because Aristotlé had also observed it centuries before.

Galileo had noticed, too, the following additional basic idea: In the case of levers and inclined planes, equilibrium is determined not alone by the weights involved but by their possible approach to or recession from the center of the earth. Thus, if any system of connected weights is so arranged that none of them can descend, there can be no motion of the other weights and the system is in equilibrium. Equilibrium therefore is established when the center of gravity of the entire system is as low as it is possible for it to go. This led Galileo to the conclusion that equilibrium existed when

$$W_1h_1+W_2h_2+W_3h_3=0$$

Essentially, Galileo's method of determining whether or not a system was in equilibrium was as follows: He posed the question, Can any work be done by gravity? If not, then the system is in equilibrium. The criteria of other investigators usually were based on the statical moments about some center of rotation.

John Bernoulli was the first to make a general application of the principle of virtual displacements. He applied it to the solution of the suspended cable. He decided that the final or stable form of such a suspended cable would be that which was attained "when all that can happen has

happened," or when the center of gravity of the entire cable was at its lowest possible position. Therefore, the principle of virtual displacements was simply a statement of a fact which everyone knew instinctively was true but which had not been made use of, namely, that heavy bodies under the influence of their own weight move downward or that there is motion only when work can be performed by gravity. If the principle is applied to force systems other than those where gravity alone acts, then displacements must take place only in what is commonly called the positive direction. Positive work must be done or displacement must take place only in the direction of the applied force.

After Galileo came a period during which his students and followers continued his work, carrying on additional experiments to prove the truth of the fundamental laws which had been developed and also formulating and simplifying the statements of the principles. These men did much writing and published many books on the subjects which had previously been investigated, such as freely falling bodies, pendulums, and projectiles. However, Galileo's theory that the parabola was the path of the projectile was accepted until 1740 when it was shown by Robins to be incorrect.

The Concept of Force Matures.—The Science of Mechanics had now progressed so that there was some understanding of the ideas which Newton later formulated into the laws of motion. Also it was now generally recognized that a force was necessary to put a body in motion, but it was not known that it did not require a force to maintain motion. The first law of motion was only partially known at this time.

The second law of motion was recognized for constant parallel forces, but the laws which govern the motions of a number of bodies which act on each other were still to be discovered. During his study of the projectile problem,

Galileo had become convinced that the velocity of projection and the velocity of the projectile caused by gravity did not affect each other when combined to produce the resultant velocity of the projectile. This conclusion, however, was based on the assumption that gravity always acts along lines which are parallel to each other, the curvature of the earth being disregarded. When such problems as the motions of the planets in their orbits were studied, trouble was encountered. Finally the idea of central forces keeping the planets in their paths began to develop. But where could such forces come from? The answer to this was Newton's contribution. Another cause of difficulty was the inability to handle variable forces mathematically, as such a technique was not successfully developed until Newton invented his fluxions.

The concept which grew into the third law of motion was still a long way from its final form, as is illustrated by some of Galileo's statements in *Discorsi*. He says: "There are two kinds of resistance in a movable body, one internal, as when we say it is more difficult to lift a weight of a thousand pounds than a weight of a hundred; another respecting space, as when we say it requires more force to throw a stone one hundred paces than fifty." From these conditions he seems to have concluded that the change of momentum due to percussion is infinite, because he says: "There is no resistance however great which is not overcome by a force of percussion however small."

Principia Clarifies Concepts of Force and Motion.—The publication of Newton's *Principia* in 1687 clarified and simplified such principles of Mechanics as had been proven, but it did not introduce any original or new inductive discoveries. It was a truly great work—probably one of the most important single accomplishments of mankind in its far reaching influence—but there was still much to be accom-

plished in the improvement of mathematical detail and the development of the techniques of solution before Mechanics could attain the effectiveness and utility which it possesses as an essential part of our twentieth-century living.

The third law of motion as stated by Newton (action and reaction are equal and opposite) was satisfactory for cases where the action between bodies was direct. But when the action was indirectly applied, as in the compound pendulum, or where several masses or bodies were connected by restraining cords, links, or otherwise, it still needed some explanation. The discussions of these problems by Descartes, Newton, Huygens, and others brought out many queer and unusual statements.

In 1673 Huygens in his *Horologium Oscillatorium* published a solution which was not generally accepted as correct at that time. The theory presented by Huygens was: "If any system of weights is set in motion by the force of gravity, they cannot move so that the center of gravity shall rise higher than the place from which it descended." Assuming the validity of this statement, Huygens was able to show that the center of gravity of any frictionless system would always attain the level of its starting position. With this principle he was able to solve the problem of the compound pendulum to his own satisfaction, but his solution brought forth considerable criticism from others. Such ideas as the following were presented: "In a compound pendulum the sum of the velocities of the component weights is equal to the sum of the velocities which they would have acquired if they had been detached pendulums." Another presented was: "The time of vibration of a compound pendulum is an arithmetic mean between the times of vibrations of the weights moving as detached pendulums." Huygens was able to discredit these theories by showing that the application of either of them could result in the center of gravity rising above its original position.

These statements created considerable uncertainty and confusion about the validity of the third law of motion when it was applied to cases involving indirect action. In order to clarify the interpretation of the law, James Bernoulli submitted the question to the mathematical world in 1686. After a considerable period of discussion an agreement was reached in the form of a statement that the actions and reactions of the various particles of a compound pendulum were distributed according to the recognized laws of statics. Many strange ideas were expressed during the course of the debate, such as the following which Leibniz contributed[6]: "The same force was required to raise one pound four feet as to raise four pounds one foot." Apparently the conclusion arrived at was not universally satisfactory because there was more or less discussion on the subject until 1743, when D'Alembert proved that the original solution presented by Huygens was correct and nothing important had been brought out by the years of debate.

After the discovery and formulation of the fundamental principles of Mechanics had been completed, there remained only the tasks of turning this accumulated knowledge into usable form and the perfection and development of the technique of the application of the principles. Newton's *Principia* was the first great systematic treatise on Mechanics. It was a task without precedent in the history of the world at the time but, as we have seen, his method was entirely geometrical and therefore difficult to use.

As the study of the Science of Mechanics proceeded and knowledge of the principles became more widespread, it grew more and more evident that mathematical analysis was a much more satisfactory tool then geometry for the further development of this science. Euler led in this phase. His first contribution along analytical lines, *Mechanica Sive Motus Scientia*, appeared in the Transactions of the Academy

[6] *History of Inductive Sciences*, by Whewell, p. 360.

of Sciences of St. Petersburg in 1736. As a preface to this article he explained that the work of Newton and others was satisfactory but was not readily applicable to new problems. Euler solved many problems by the analytical method—so many in fact that they continued to appear in the publications of the Academy of St. Petersburg for twenty years after his death, which occurred in 1783.

Mathematical Methods Displace Geometrical Procedure. Shortly after Euler's death, Lagrange published his *Mécanique Analytique* (1788). As we have seen, this treatise was a radical swing away from the geometrical method to the entirely mathematical solution with no diagrams at all. Today, while our solutions are now preponderantly mathematical, we have learned that graphical methods do possess marked advantages in certain types of problems. Furthermore, we have developed a technique whereby we are able to combine the two methods to great advantage. The modern writer has also learned that by the judicious use of diagrams he is able to transmit to the reader a clearer concept of force action than by any other method so far introduced.

With the presentation of Lagrange's *Mécanique Analytique*, the Science of Mechanics was on its way toward the development of a technique which would make it one of the more powerful tools for the vast and important technical developments and tremendous physical construction which the engineering profession has contributed to the betterment of human living during the past 60 to 100 years.

The further development of Mechanics which has taken place since Lagrange (1813) has been almost entirely devoted to clarification, improvements in notation, mathematical procedure, technique, and arrangement rather than to the introduction of any important new principles.

When we examine our present-day books on Mechanics and compare them with the books in use a half-century ago,

we find a great difference in both appearance and lucidity; yet fundamentally the material presented is much the same if the reader is sufficiently well informed in the subject to recognize it. It is doubtful if any other basic science contains so little fundamental material or so few concepts and yet is so difficult to master.

The more recent developments just mentioned have materially increased the usefulness of Mechanics and made it the servant of a vastly larger group of men who would have been incapable of using it in the form in which Newton and his followers left it. In its original form it would have remained an instrument of the mental aristocracy of the mathematical and scientific world rather than one of the basic tools of the practical engineer.

Part III

Mechanics of Materials

CHAPTER 11

Pre-Elastic Development

Beginning and Function of Mechanics of Materials. Mechanics of Materials has been defined as a science created on a base of certain fundamental principles of Mechanics and experimentally determined physical information, plus a considerable amount of common sense and engineering judgment.

For the first recorded work in this phase of physical science we must return again to the writings of Galileo. He appears to have been the first to become sufficiently interested in the causes of structural failures to attempt to arrive at a solution of these difficulties through mathematical reasoning and organized experimentation.

Since the science of Mechanics of Materials is dependent on some of the basic principles of Mechanics and experimentally determined facts, its beginning had to await the attainment of a definite maturity of Mechanics and of the technique of physical experimentation. The tree had to have a well established root system and a sizable trunk before its branches could reach out far from the supporting base.

Mechanics of Materials in some one or more of its different phases is a necessary tool for the designer of all of man's important structural accomplishments. It is an exceedingly difficult task to attempt to join into a well woven fabric the multitudinous contributions of the vast army of workers in this field in the more than three centuries since Galileo first attempted to find out why some beams failed and others did not; and the task becomes more difficult as we move on down through the years to the last few decades during which the more incidental refinements have been introduced.

The object of this book is not to review all of the many technical advances made in the science of Mechanics of Materials down to the present time, but it is merely to attempt to point out such basic items as have had an important influence on the over-all growth of the science. If we fail to give credit where it is due, such an error may be accidental or it may simply mean that the perspective of the picture varies when viewed from different directions. The painter can only paint the picture as it appears to him and with such colors as are on his palette at the time.

Importance of Experiment and Research.—Addressing the Silver Anniversary Forum on the Future of Industrial Research on October 5, 1944, Robert P. Patterson, Under Secretary of War, said: "There is a great voice in the world today, the voice of science and technology. It is a voice heard since ancient times but never until today has it spoken with such authority, have its words been so filled with promise, has it been listened to with such hope, and in no country in the world does the voice speak so eloquently as in our own.

"If I were to add to our four freedoms of the Atlantic Charter, I might suggest a fifth freedom of inquiry, experiment and research.

"While important scientific advances are not often made in attic, cellar and barn, as was the case not so long ago, we must not permit the precious stream of discovery flowing from smaller industry and smaller educational institutions to be dammed up by neglect."

Speaking before the same group, Dr. Thomas Midgley, Jr., President of the American Chemical Society, had this to say of what he calls the scientific process: "What is this scientific process? Indeed, all too often the term is used to glamourize simple common sense and then again it is used by charlatans to give respectability to what otherwise obviously would be

in conflict with common sense. A better definition is desirable. To my mind the basis of the scientific process is the reproducible experiment.

"Facts are still, and probably always will be, determined by vote; quite as the College of Cardinals determined the number of feathers in the Archangels' wings just a few centuries ago. To the modern scientifically trained mind this process seems somewhat ridiculous; but was it? The number had to be determined for the benefit of painters, sculptors and architects, and how else could it have been done? Indeed, the same process is in use today when groups of scientists gather to hear discussions of controversial subjects. There are, however, two points of basic difference. Whereas the number of feathers was decided by majority vote, in science we require a practically unanimous vote for establishing a fact. The second point is the type of evidence required by the voter to influence his decision. Revelations, dreams, supernatural authority are now out, and even logic is of secondary importance to the reproducible experiment.

"Mathematics is the only branch of science which claims exemption from this rule. Two thousand years ago, mathematics passed from the realm of the experimental to the Utopia of pure logic."

In an address in 1944, Dr. James B. Conant, President of Harvard, said the future of the physical sciences depended on the "number of really first class men" that can be turned out by our educational institutions.

Why are these men so interested in maintaining and protecting the unhampered growth of scientific research? Because it is only by the fruits of such research that we can hope to maintain our present standard of living and world leadership. These are practical men who wish to retain our advantageous position in world affairs and, if possible, make it more secure.

This, then, is the position which science in general, and scientific research and Mechanics in particular, had attained in 1947. This is the long and difficult climb it has made since Friar Bacon, Galileo, and their followers stood at the base and looked toward the mountain top before they began the hesitating and devious ascent.

Relation of Mechanics of Materials to Mechanics.—During the entire period of incubation, adolescence, and maturity Strength of Materials or Mechanics of Materials has been directly and inseparably associated with the basic Science of Mechanics. We find this in the college catalogs, in our search for historical data, and in the practical every-day relationships brought about by the construction of the things which we in America and the rest of the civilized world are today making in so varied and such vast quantities. Without this father-and-son relationship of the two sciences, Friar Bacon's thirteenth-century prophecies could never have been realized and, lest we forget and think too well of ourselves, it would have been impossible to produce our modern war-time engines of destruction. It is the intimate day-to-day harmonious living together of these two branches of science which is, in a large measure, responsible for the great progress made in the improvement of our creature comforts and in giving us greater leisure for the development of our esthetic abilities.

As we look back at the tools and structures which man built previous to 1800, we are somewhat surprised to realize that, even at such a late date, little knowledge of the strength of materials was actually needed by the builders of the world. True, certain features connected with large cathedrals and other large structures, such as vaults, arches, and flying buttresses, required some knowledge of materials. Also, certain sizable bridges and the rigging of large sailing vessels presented some challenge to the builders who wished to

continue in business. But before 1800 the materials used and the customary proportions which had been developed almost entirely by the "cut-and-try" method generally proved sufficiently accurate to keep most designers and builders out of serious difficulty. The usual procedure was to make large structures geometrically proportional to successful smaller structures—a practice which can produce many surprises, as most engineers today well realize. Almost up to the nineteenth century the stimulus to learn about the things which were essential to put such construction on a scientific basis was not very great. The practical necessity had not arisen, and the incentive was only that of the scholar to uncover the scientific facts—not a sufficiently urgent need to produce any considerable effort.

Galileo's Cantilever-Beam Experiments.—In our search for the first bit of tangible evidence which leads in the right direction, we come again to the work of that wise and able man of Italy, Galileo, who contributed so much to the founding of Mechanics.

Galileo, in the second dialogue of the *Discorsie Dimostrazioni Matematiche* (Leyden, 1638), or in its English translation in Thomas Salusbury's *Mathematical Collections and Translations* (London, 1665, Vol. 2, p. 89), wrote what appears to be the first important contribution dealing with Strength of Materials. In this discussion Galileo presents seventeen propositions about the fracture of rods, beams, and hollow cylinders. This paper's chief value is due to its great historical importance. Galileo makes the incorrect assumption that the fibers of a strained beam or rod are inelastic and cannot therefore be stretched. He discusses the bending of a cantilever beam, the fixed end of which is built into a vertical wall. The beam is strained by its own weight and the weight of the concentrated load attached to its free end. Galileo thought of a beam as being a rigid body, except for its rota-

tion about an axis in the plane of rupture which is at the surface of the supporting wall.

Even with his incorrect assumptions (he had no knowledge of any law connecting displacements with the forces causing the displacements or any hypotheses which might be used to formulate such a law), Galileo was able to work out a satisfactory design for a cantilever beam of uniform strength with a rectangular cross-section. Because he had no knowledge of any stress-strain relationship, he assumed that a solid was inextensible or incompressible and that there was a uniform stress distribution over the entire cross-section until fracture occurred. For a rectangular section, which was the design chiefly used during his time, Galileo's solution was sufficiently accurate for his purpose.

As the result of his experiments with cantilever beams, Galileo arrived at the following two conclusions:

The resistances of the bases to fracture of similar prismatic beams are as the squares of their corresponding dimensions.

Among an infinite number of homogeneous and similar beams, there is only one for which the weight of the beam is exactly in equilibrium with the resistance of the beam at the plane of fracture. Longer beams will break and shorter ones will not.

The cantilever beam became known to later workers in the science of Strength of Materials as Galileo's problem. Galileo's discussion of the cantilever of uniform strength—or, as he called it, a solid of equal resistance—started a considerable controversy which was carried on by the scientific workers of the period for about seventy years. P. S. Girard in his *Traité Analytique de la Résistance des Solides*, which was published at Paris in 1798 and the German translation of which appeared in 1803, discusses this controversy.

Experimental Base of Mechanics of Materials.—Galileo's problem, although related to the growth of the unknown

theory of elasticity, was primarily a beam-flexure problem. The continental workers were attempting to solve this problem without possessing any conception of elasticity which could eventually bring them to the elastic curve, while their English contemporaries were at the same period busily engaged in discussing hypotheses as to the nature of elastic bodies without seeming to arrive at any satisfactory conclusions. Robert Hooke, however, in his *De Potentiâ Restitutiva* (London, 1678), states that eighteen years before, in 1660, he had discovered the theory of springs; but, because of his desire to obtain a patent on a particular application of the theory, he did not make his discovery known. Hooke, like many other inventors, did not realize that his discovery of the relationship which is now known as Hooke's Law unlocked the door to a new science. Hooke's Law became not only the corner-stone but actually the base upon which man has built one of his most utilitarian sciences—a science which has made possible most of the structural advances of the last century or more. Hooke's spring theory was stated in *De Potentiâ Restitutiva* in the following manner.

"Take then a quantity of even-drawn wire, either Steel, Iron or Brass, and coyl it on an even Cylinder into a Helix of what number of turns you please, then turn the ends of the Wire into loops, by one of which suspend this coyl upon a nail and by the other restrain the weight that you would have to extend it, and hanging on several weights observe exactly to what length its own weight doth extend it, and you shall find that if one ounce, or one pound, or one certain weight doth lengthen it one line or one inch or one certain length, then two ounces, or two pounds, or two weights will extend it two lines, two inches or two lengths and three ounces, pounds or weights, three lines, inches or lengths and so forwards."

By a spring Hooke did not mean just the ordinary conventional spring but meant any "springy body" whatever;

and he attempted to define a "spring" by referring to a long list of materials which he considers are of a "springy" nature.

Galileo's problem had created interest in the behavior of materials, but Hooke's law was the basic experimental foundation stone upon which man was to build, during the next three centuries, a Science which would permit him to create and destroy structures so large and powerful that Archimedes' boast that he could lift the world if he had something to stand on does not seem a too ridiculous jest.

Robert Hooke, Catalyst.—Robert Hooke, M. A., M. D., F. R. S. (1635–1702 or 03), was recently honored by the publication of a portion of his diary. This interesting book covers the period of his life between 1672 and 1680. It was edited by Henry W. Robinson, librarian of the Royal Society, and Walter Adams, and was published in 1935 on the Tercentenary of Hooke's birth.

Hooke was a genius with many talents, having made contributions to biology, geology, astronomy, mechanics, and physics. He was the inventor of the balance spring and anchor escapement for watches, the wheel barometer, and improvements in the microscope, telescope, and air pump; he suggested the freezing-point as zero on the thermometer scale; and he also carried on professionally as an architect, as a surveyor, and—in later years—as a physician, even though he himself had much ill health, which is now thought to have been caused by a chronic sinus infection.

Hooke was born at Freshwater, Isle of Wight, where his father was curate at All Saints Church. As a child he was weak and apparently had much digestive trouble for he lived on milk and fruits for seven years. His father had hoped to have him study for the ministry but gave up the idea when the boy's health did not improve. Because of his frail health he received little formal education but early developed inventive talent and the will for self-education.

When Hooke was thirteen his father died and he was apprenticed to Peter Lely, a well established and fashionable London portrait painter. He soon gave up the apprenticeship and entered Westminster school where Dr. Busby, the headmaster, encouraged him to study mathematics. He is credited with mastering the first six books of Euclid in a week. His mathematical ability was then turned to Mechanics, and his general education was rounded out with Latin, Greek, and music.

In 1653 he became a chorister at Christ Church, Oxford, and assistant in chemistry to Thomas Willis who soon passed him on to Dr. Robert Boyle—a most important event in Hooke's career since he and Boyle became lifelong friends. Hooke is credited with being somewhat of a mathematical tutor to Boyle to the extent of Euclid's Elements and Descartes' Philosophy. His association with Boyle and a group of Boyle's friends who had formed a philosophical club at Oxford was a most valuable experience. This group included such now-famous names as Christopher Wrenn, Thomas Millington, and others who at the time were mostly between twenty-five and thirty-five years of age. Hooke began to attend the meetings of the club in 1655 at the age of twenty.

As Boyle's assistant, Hooke's duties included the construction of apparatus for experimental work. He soon became known as the most expert mechanician of his time but apparently, even though he possessed great mathematical ability, he was not so skilled in stating the results of his work in mathematical language.

It was while working with Boyle that he invented the balance spring for watches and made other improvements in time-recording instruments which he claimed would have been a great help to navigators by enabling them to determine longitude at sea. Unfortunately Hooke's personality was a bit on the jealous and suspicious side. When he told

Boyle and others about his balance-spring invention, he did not tell them of his longitude-determining method. These friends drew up a patent which gave him full credit for the time-measuring improvement and the major share of any profits that might be realized; but the patent reserved their right to all benefits which might occur through any future improvement that might be made. To this Hooke would not agree, and so the proceedings were dropped. It has been suggested that Hooke's method of determining longitude had not been perfected at the time and he therefore decided to wait until such time as it was before entering into any agreement with his friends. As was the case with many of his ideas, he never got around to finishing it. His balance-spring and anchor-escapement inventions made Hooke an important personage in horology.

On November 12, 1662, at the age of twenty-seven, Robert Hooke became Curator of Experiments for the Royal Society of London; and two years later on July 27, 1664, his salary was placed at £80 per year, supplemented by lodgings at Gresham College, the official meeting place of the Society. On March 20, 1665, he was made Professor of Geometry at Gresham.

As curator of experiments it was his duty to perform experiments before the Society at its weekly meetings. He is now generally credited with keeping the Society alive through his personal efforts and the interest created by his experiments. His position as curator was made more difficult because it was necessary to keep up the interest of a divergent group of personalities—some serious workers and others merely seeking amusement. This made his work more or less superficial. His experiments had to be spread over a wide field if they were to serve the purpose for which they were intended—stimulation of scientific interest.

There is little doubt that if Hooke had been permitted to spread his talents a little less thinly his achievements would

have been more noteworthy; but, then, he would not have been Robert Hooke the "catalyst."

Although he was forced to discontinue his experimental work before the Society several times for such reasons as the plague of 1665, the London fire of 1666, and the demands placed on him by public service, he continued as Curator of the Royal Society until his death on March 3, 1702 or 1703.

Mariotte Locates the Neutral Axis.—At about the time at which Hooke was doing his investigating in England, the French were also endeavoring to establish the relationship between force and deformation. In 1686, at Paris, Mariotte in his *Traité du mouvement des eaux* gives the results of experiments made in 1680, which demonstrated that Galileo's beam theory does not agree with the behavior of beams during bending. Mariotte observed that some of the beam fibers were stretched and some were compressed before rupture of the beam occurred. He therefore assumed—without proof—that half were stretched and half were compressed, or that the line which we now call the neutral axis was at the center of the cross-section of the beam. Mariotte also decided that the resistance of a beam to bending was due to this extension and contraction of the fibers. Mariotte's rejection of Galileo's theory of the rigid body appealed to C. W. Leibniz; and in *Demonstrationes novae de Resistentiâ solidorum*, in *Acta Eruditorum*, published in July, 1684, he attempted to settle the Galileo-Mariotte disagreement by the dictatorial announcement that there was always flexure before a beam ruptured and therefore some fibers were stretched and their resistance was according to Hooke's Law. However, the application of this theory to practical problems he would leave to those who had more leisure for such matters than was available to a man of his mathematical ability. This, then, was the beginning of the now accepted theory for the flexure of beams.

Disagreement Over Flexure Theory.—For approximately 160 years after Hooke established the relation between stress and strain, or until Navier presented for the first time the general equations of elasticity, Galileo's problem, the vibrations of plates and bars, and the stability of columns seem to have absorbed the energy and minds of the mathematicians who were interested in discovering the fundamental theory of Strength of Materials.

In the memoirs of the Paris Academy for 1702 will be found a paper by Varignon in which he says that it is possible to state a general formula for beams which will incorporate both Galileo's hypothesis and the Mariotte-Leibniz hypothesis. However, he failed to do so and computed the resisting moment for the cantilever by assuming the tensile forces of the fibers to be proportional to the distances from an axis placed at the bottom of the beam cross-section. This entirely disregarded Mariotte's experimental observation that some fibers were stretched and some shortened.

James Bernoulli, the earliest of the Bernoullis, in *Acta Eruditorum Lipsiae*, published in June, 1694, seems to be the first to arrive at anything of real mathematical value on the theory of beams. However, he apparently did not like his first presentation because he published a revised version in 1705 under the title *Véritable hypothèse de la résistance des Solides, avec la démonstration de la Courbure des Corps qui font ressort*, which also appears in his collected writings that were published at Geneva in 1744. In this article Bernoulli claims to have been the first to introduce compression into the bending of beams. He evidently was not well informed on Mariotte's previously published work. In this paper we find the first appearance of the idea that a section which is plane before bending remains a plane after bending, and he finally obtained the equation of the longitudinal axis (elastic curve) of the beam from the theory that the extension and contraction of the fibers caused the resistance of the beam. His

equation showed that the resistance to bending was a couple approximately proportional to the curvature of the beam. Bernoulli seems to have had trouble determining which fibers were stretched and which were compressed. In fact, he finally concludes that the same resistance to bending would be developed if all fibers were stretched, all were compressed or some were stretched and some compressed. This means that the location of the neutral axis is unimportant, a fact which today would be disputed by any student of Mechanics of Materials; thus, Bernoulli did not agree with the Mariotte-Leibniz theory that the resistance of the fibers followed Hooke's law. The over-all evaluation of Bernoulli's discussion therefore must be that it was invalid and unsatisfactory; but, like the attempts of Galileo and Mariotte, Bernoulli's work was not without value because it apparently stimulated thought which hastened the arrival of the correct solution.

Experimentalists Try to Explain Elasticity.—While the physicists and the mathematicians were trying to learn something about the basic principles of elasticity, many other more practical men were trying by experimental methods to find a method which would give a satisfactory solution for the flexure problem. The knowledge which these experimentalists obtained was of material assistance to the theorists. In addition to the work already accredited to Mariotte, which he performed in 1680, further contributions were made by A. Parent in 1702; by R. A. F. de Réamur in 1711; by B. F. Bélidor in 1729; and also by the publication in 1729 of *Introductio ad cohaerentiam corporum firmorum*, a book of considerable volume, by Petris van Musschenbroek, the latter part of which is devoted to the author's *Physicae experimentales et geometricae Dissertationes*. There is, however, a valuable historical preface in which are described the various theories previously presented by other workers.

Van Musschenbroek ridicules Bacon's explanation of elasticity, calls another metaphysical hypothesis abracadabra, and finally returns to Newton's theory of internal forces, which he considers can be determined in each case by experiment and need no metaphysical hypothesis to explain their cause. This book remained in high repute until the end of the eighteenth century.

In 1724 John Bernoulli wrote a prize essay entitled *Discours sur les loix de la communication du mouvement,* in which he attempted to explain elasticity by assuming that the elastic body contained compressible ether cells. Other unsuccessful attempts also were made to explain elasticity.

In 1731 Jacope Riccati published at Bologna a paper which described the first experimental attempt since Hooke to determine the laws which govern the behavior of elastic bodies. His results, however, had little value. Thirty years later (1761) the *Opere del Conte Jacope Riccati* was published, in which Riccati attempts an explanation of the general character of elasticity. This also was of little value scientifically, but must be mentioned because of his choice of the experimental method of approach and his desire to cast aside metaphysical hypotheses for a dynamical theory. In this respect he was somewhat like Friar Bacon, a man who had lots of ideas on how things should be done but who accomplished little of the doing himself. It seems that the world has always produced this type, who no doubt make their contribution but whose apparent indolence proves irksome to the less brilliant but more consistent, tireless workers who struggle on insistently after facts.

Euler Derives the Equation of the Elastic Curve and His Column Formula.—Leonhard Euler was born in 1707 at Basle, Switzerland. In 1727, when twenty years old, he was invited by Catherine I of Russia to the St. Petersburg Academy, and in 1741 Frederick the Great induced him to come

to the Berlin Academy. In 1766, at the invitation of Catherine II, he returned to the St. Petersburg Academy, where he died in 1783.

There seems to have been a close friendship between Euler and the Bernoullis, who came from the same town in Switzerland. There was considerable correspondence between them, which no doubt created or helped to create Euler's interest in elastic problems. In a letter to Euler in 1735, Daniel Bernoulli mentions results of his research on the period of vibration of a transverse bar; and in a letter to Euler in 1739, John Bernoulli writes of "an elastica rectangula," about which Euler had previously written. It seems that here again Galileo's problem had created the desire to learn something about the curve which the loaded beam assumed. Acting on a suggestion by Daniel Bernoulli (who had been made Professor of Mathematics at St. Petersburg Academy by Catherine I in 1725) that he had discovered in his labors that $\int \frac{ds}{R^2}$ was a minimum for the elastic curve of a beam, Euler proceeded to investigate the inverse problem of finding the equation of the curve for which Bernoulli's potential force should be a minimum. This discussion appeared in 1744 in *Methodus inveniendi lineas curvas maximi minimive proprietate gaudentes* as an appendix called *Additamentum I De Curvis Elasticis.*

From the elastic curve of a beam, Euler soon progressed to his well known column formula. This appeared in a paper written in 1757 and published in the Mémoires de l'Académie de Berlin in 1759 under the title *Sur la force de colonnes.* Historically this is one of the most important contributions to the theory of Mechanics of Materials. Also it is unique. Here we have the equation of the elastic curve of a beam and also the column formula before the basic flexure problem had been satisfactorily solved—almost two decades before that result was to be arrived at.

The Concept of Elastic Stability.—In the 1771 Mémoires de l'Académie de Berlin there is a paper by Lagrange in which he discusses the force required to bend springs; and in the Mémoires of the Royal Society of Turin for 1770–1773 there appears additional work on columns by Lagrange. Euler and Lagrange soon learned that up to a certain load a given column did not bend, but when that load was exceeded bending began. Here we have the birth of the idea of elastic stability. In this paper Lagrange first developed the equation for a cylindrical column and then gave the theory for the small bending of columns with other cross-sections.

In this early work on the elastic curve of beams and columns the quantity now known as EI was called the "moment of stiffness" and was assumed to be a characteristic of the column or beam. Euler proposed to obtain this quantity experimentally by supporting the beam or column as a cantilever and applying a concentrated load at the free end, and computing the "moment of stiffness" (*moment du ressort* or *moment de roideur*) from the measured deflection.

Coulomb's Flexure Theory; Shearing Deformation. In the 1773 *Mémoires par divers Savans*, which were published in 1776 by the Paris Academy, there is a paper by Charles Augustin Coulomb (1736–1806), a military engineer, physicist, and mathematician of considerable prominence, which is entitled *Essai sur une application des règles de Maximis et Minimis à quelques Problèmes de Statique, relatifs à l'Architeture.* In this paper we find that Coulomb was the first to realize that the horizontal summation of forces at any section of a beam, as well as the moment, must be considered for equilibrium. Knowing that for equilibrium the horizontal summation of forces must be zero, he was able to determine the correct position of the neutral axis, or that it passes through the centroid of the cross-section. Coulomb's theory for the bending of beams is the most exact of all those which

are based on the assumptions that the stress in a bent beam is caused entirely by the extension and contraction of the longitudinal fibers of the beam and that these stresses are proportional to the strains or follow Hooke's Law. Having correctly located the neutral axis, he was able to obtain the resisting moment of the internal forces. There is also a discussion in the paper about the kind of deformation which we now call shearing deformation. Coulomb was the first to recognize it and expressed the belief that rupture or failure occurred when the shear exceeded a certain amount.

The great elastician Saint-Venant thought so highly of this paper that eighty-three years later (1856), in the Journal de Mathématiques (Liouville), he says that in Coulomb's paper "one finds presented almost all of the bases of the theory of stability of structures." He also says that Coulomb changed the beam problem from an elasticians' problem (elastic curve) to an engineers' problem (resistance to flexure). The paper is discussed in both Todhunter and Pearson's *A History of the Theory of Elasticity* (1886–93) and A. E. H. Love's *Mathematical Theory of Elasticity* (Third Ed., 1920). Many others had attempted to solve the problem of the flexure of a beam since Galileo first labored with it, but their theories and solutions were found wanting. Coulomb must be given credit for the first satisfactory practical solution. This paper by Coulomb is one of the most important contributions to the growth of the Science of Mechanics of Materials. It seemed to generate new interest and soon many other papers and books began to appear.

The First Torsion Theory.—In the Histoire de l'Académie for 1784, published in 1787 at Paris, there appears another paper by Coulomb entitled *Recherches théoriques et expérimentales sur la force de Torsion et sur l'élasticité des fils de métal.* This paper seems to be the first recorded discussion of torsion and apparently is what Saint-Venant refers to on

pages 331, 340, and 341 of his work *Torsion* when he mentions Coulomb's ancient theory of torsion. Coulomb does not express his torque in terms of the elastic rigidity but makes it proportional to the moment of inertia of the normal section about the longitudinal axis of the rod or wire.

Further Discussion of the Flexure Theory.—In 1782 Giordano Riccati, son of Jacope, in a paper on *Delle vibrasioni sonore dei cilindri*, which was published in Verona, apparently had not accepted or heard about Coulomb's solution for flexure of beams, because he places the neutral surface at the bottom of the beam. This fact may be evidence of the slow passage of knowledge or only of Riccati's inadequate scholarship.

In an *Essai théorétique sur les vibrations des plaques élastiques rectangulaires et libres*, which was written for the Academy at St. Petersburg in 1788, James Bernoulli attempts to obtain the equation for the vibrations of a plate by an adaptation of Euler's method for bells, but he gets into difficulties which he cannot master.

In order that the relationship of the various Bernoullis may be understood, we present the following data:

- Nicolaus (1623–1708) (A merchant of Basel)
  - John (1667–1748) (Discourse on Motion)
    - Nicolas (1695–1726)
    - John (1710–1790)
      - John (1744–1807)
      - James (1759–1789)
    - Daniel (1700–1782) (Vibration of a beam)
  - James (1654–1705) (Elastic curve)

In 1798 *Traité Analytique de la Résistance des solides, et des solides d'égale Résistance* by P. S. Girard was published in Paris, and in 1803 a translation into German was published at Giessen, Germany. The book contains a historical sketch covering the contributions of Galileo, Leibniz, Mariotte, Euler, and Lagrange to the Science of Mechanics of Materials; and also the results of considerable experimental work carried on by the author. Girard seems a bit confused about the theory of beams. He suggests that Galileo's hypothesis of non-extension of the fibers may be true for some bodies such as stone and minerals but the Mariotte-Leibniz theory is true for fibrous materials. He also appears not to have heard of or not to have accepted Coulomb's location of the neutral axis. He agrees with James Bernoulli that the position of the axis is unimportant.

Evaluation of the Work of English, German, and French Writers.—Immediately following the turn of the century, several English and German textbooks were published. None of these books offered anything of value for the advancement of the science. As Todhunter and Pearson point out in their *History of Elasticity and Strength of Materials*, "nothing shews more clearly the depth to which English mechanical knowledge had sunk at the commencement of this century."

One of these books, *On the Power of Machines* (1803) by John Banks, who calls himself a "lecturer on philosophy," devotes considerable space to the results of experiments on the strength of oak, fir, and cast iron, but it still uses Galileo's hypothesis and the incorrect position of the neutral axis.

The *Treatise of Mechanics Theoretical, Practical and Descriptive* (1806) by Olinthus Gregory also discusses strength of materials and reproduces the whole of Galileo's results, despite the fact that they had been shown to be erroneous by Mariotte over a hundred years earlier.

At Berlin in 1808, Eytelwein published his *Handbuch der Statik fester Körper*. This book contains a bibliography of the many important earlier contributions and many experimental results taken from the books of Petris van Musschenbroek and P. S. Girard. There is no new theory in the book, but the author does recognize the correct position of the neutral axis. In his analysis of the flexure problem, however, he falls back on the old theory which rejects compression, evidently because the newer analysis appeared too difficult.

Here were three books which certainly had little to commend them. Nevertheless, they apparently were acceptable to great numbers of people.

A bit later in London, Thomas Tredgold published *The Elementary Principles of Carpentry* (1820) and *A Practical Essay on the Strength of Cast Iron* (1822). Both books attempt to explain theory but, as indicated by the titles, they cannot be expected to be very exacting in their theoretical demonstrations. In the later book Tredgold attempts to comment on Newton's fluxions. Of this Todhunter and Pearson say: "Such is the scientific capacity of the man whose works remained for years the standard textbooks of English engineers."

In France, A. Duleau was the author of *Essai théoretique et expérimental sur résistance du fer forge*, published at Paris in 1820. This was an essay resulting from experiments made in connection with the construction of an iron bridge over the Dordogne, which received the approval of the Academy of Sciences. Duleau accepted Coulomb's ideas but apparently misinterpreted them.

Returning to England we find books and papers by Peter Barlow and E. Hodgkinson. In 1817 Peter Barlow wrote an *Essay on the Strength and Stress of Timber*, which passed through several editions. This contains a historical sketch which follows Girard's closely and also includes comments on the work of Euler and Lagrange. Barlow apparently

thought that Leibniz' placing of the neutral axis at the lower surface of the beam was still the position accepted by the investigators of his time, even though it had been correctly located by Coulomb and also by G. B. Bülfinger in *De solidorum Resistentia Specimen, Comentarii Academiae Petropditanae* (1729). Later (1837), in an edition entitled *A Treatise on the Strength of Timber and Cast Iron, Malleable Iron, and Other Materials,* Barlow accepts Coulomb's principle which he calls Hodgkinson's simply because Hodgkinson had criticized his (Barlow's) own earlier work.

E. Hodgkinson, an experimentalist of some ability, wrote two papers for the Literary and Philosophical Society of Manchester. The first, in 1824, was entitled *On the Transverse Strain and Strength of Materials*. The chief merit of this paper is that after a rather circuitous discussion Hodgkinson arrived at the place reached by Coulomb fifty years earlier. However, the paper did cause sufficient discussion to firmly establish the correct position of the neutral axis among the English at least, as Eytelwein's *Handbuch der Statik fester Körper*, published at Berlin in 1808, had done in Germany.

Hodgkinson's second paper, read in 1830 and published in 1831, was entitled *Theoretical and Experimental Researches to Ascertain the Strength and Best Form for Iron Beams*. This paper is chiefly occupied with tests on T-shaped cast-iron beams which attempt to show the physical properties of the metal. It is chiefly of interest because it brings out the difference between the physical properties of cast iron and malleable iron. Hodgkinson seems to have had exceptional experimental ability for the time. Had he lived later, he might have made a more prominent place for himself.

This brief resumé of the much used books of the first three decades of the nineteenth century shows that no outstanding authors were represented. All the books were more or less written for the practical worker rather than to advance the Science. These books might possibly be considered to be

the forefathers of our much more accurate and scientifically correct present-day handbooks—tools for the practical engineers.

The English writers seemed to be especially weak. There apparently were, at this period in England, no great mathematicians who were interested in the growth of the Science of Mechanics of Materials.

Young's Modulus.—Retracing our steps a few years we find that in 1802 Thomas Young published in London *A Syllabus of a Course of Lectures on Natural and Experimental Philosophy*. Part of this syllabus deals with some of the general properties of matter. Young says on page 39: "The strength of materials employed in Mechanics depends on the cohesive and repulsive forces of their particles. When a weight is suspended below a fixed point, the suspending substance is stretched, and retains its form by cohesion; when the weight is supported by a block or pillar placed below it, the block is compressed, and resists primarily by a repulsive force, but secondarily by the cohesion required to prevent the particles from sliding away laterally. When the strain is transverse both cohesion and repulsion are exerted in different parts of the substance." He then continues with some discussion about transverse forces and the position of the neutral point. From this discussion we learn that Young assumed different moduli for compression and tension, and that he had no clear idea about the location of the neutral axis.

In 1807 Young's lectures entitled *A Course of Lectures on Natural Philosophy and the Mechanical Arts* were published in two volumes. He includes a very good bibliographical summary of the literature published during the previous century.

Dr. Young's definition of his now well known modulus is stated as follows: "The modulus of the elasticity of any

substance is a column of the same substance capable of producing a pressure on its base which is to the weight causing a certain degree of compression as the length of the substance is to the diminution of its length." This is not at all a comprehensible definition; it seems almost as if an attempt were being made to produce as confusing a statement as possible. The definition by Dr. Young does not represent the same quantity which we now refer to as Young's modulus of elasticity. Unfortunately, clearness of expression was not one of the attributes of this distinguished scholar. So able a student as Dr. Whewell says[1] of Dr. Young's work: "I would gladly have given a section on the strength and fracture of beams had there been any mode of considering the subject which combined simplicity with a correspondence to facts."

Young Names the Neutral Axis.—Even though Young does not seem to have had a clear conception of the position of the neutral line, his constant use of the term neutral line or neutral axis no doubt led to its general adoption. He was also the first to treat shear as an elastic strain. Unfortunately, inability to express his thoughts in clear logical language was a handicap which Dr. Young never overcame and which detracted much from his personal prestige as a philosopher. The average student will obtain little from the study of his lectures. Young's discovery of the modulus of elasticity was the foundation stone of practical elasticity; it was a basic contribution needed for the growth of the Science of Elasticity.

Dr. Young, Gentleman Philosopher.—Thomas Young was called by his biographer, Frank Oldham of St. John's College, Cambridge, the greatest natural philosopher of the early nineteenth century. He was born June 13, 1773, at Milverton in Somerset, a short distance from Bridgewater,

[1] *Elementary Treatise on Mechanics,* Second Edition (1824), p. 10 of Preface.

and was the oldest of the ten children of his Quaker parents. His early Quaker training was visible throughout his life, even though in his later years he accepted one of the sacraments of the Church of England.

In *Disciples of Æsculapius*, Volume II, Sir Benjamin Ward Richardson gives a picture of this strange combination of philosopher, student of languages, scientist, practicing physician, and society man, who was responsible for the second basic physical relationship so essential for the further development of Mechanics of Materials.

Young first attended school at Bristol, but conditions there were not congenial and he was soon transferred to an academy at Compton. There, under the guidance of Mr. Thompson, an uncle, he made such rapid progress that when he left the school at the age of fourteen he was quite proficient in elementary mathematics. Moreover, assisted by his already well developed memory, he had made considerable progress in the Greek, Latin, French, Italian, Hebrew, Persian, and Arabic languages, as well as having demonstrated unusual skill in the practical arts of bookbinding, color mixing, copper-plate making, lathe work, lens grinding, and construction of scientific instruments.

The choice of a profession proved difficult because of his scholarly aptitude, but he finally decided on medicine, being influenced no doubt by the great success of the renowned Dr. Brocklesby, who was his mother's uncle. Before arriving at a decision, he served as a tutor in the home of Mr. David Barclay of Youngsbury, Hertfordshire. While thus engaged he had continued his study of mathematics and added the sciences of botany, geology, and etymology and the Egyptian language to his already diversified interests.

The advanced age of Dr. Brocklesby—almost seventy—probably was the deciding factor in his choice of medicine as a career. He went to London where, in the course of his medical training, he attended lectures and demonstrations

given by such able men as Wilson, Baille, and Cruishank on anatomy; John Hunter on surgery; Clark and Osborne on obstetrics; Sir J. W. Smith on botany; and Dr. John Latham on medicine.

His friends who had earlier tried to interest him in the law or church as a career now renewed their efforts in favor of a law course at Cambridge, with the added inducement of an assistant-secretaryship to the Duke of Richmond, then Master-General of Ordnance, but their efforts were again unsuccessful and he returned home after completing his medical work in London.

In 1793 at the age of twenty he sent his first paper, *Observation on Vision,* to the Royal Society. As a result he was made a Fellow of the Royal Society in 1794, when only twenty-one years of age. Next came study at Edinburgh under Black and Monroe and further work at Göttingen, where in 1795 he received the degree Doctor of Physic, Surgery, and Man-midwifery. After visiting the Harz Mountains, Leipsig, Jena, Dresden, Berlin, Hamburg, and Brunswick, he returned to Cambridge for three more years of study and the degree of Doctor of Medicine which permitted him to begin the practice of medicine.

During Young's stay at Cambridge, Dr. Brocklesby had died and left him his library and a comfortable fortune sufficient to provide for all future needs. Young was now free to devote all of his energy to such scholarly pursuits as he chose.

In 1802 Dr. Young established a home in the best West End section of London, with the intention of practicing medicine, but he was soon induced to travel in France with the Duke of Richmond and Lord George Lennox. He remained in Paris for some time, becoming acquainted with many of the famous scholars at the Paris Institute. However, his stay in Paris had to be terminated because Napoleon's increasing power was making relations between the

French and English difficult. Back in London again he was made Professor of Experimental Philosophy at the Royal Institution of Great Britain, and in 1804 he became foreign secretary of the Royal Society.

Dr. Young apparently was not too successful as a lecturer, for he soon resigned his position at the Royal Institution and then proceeded to have his lectures published as *A Course of Lectures on Natural Philosophy and the Mechanical Arts*. Commenting on this appointment and the resulting publication, Young's biographer, Frank Oldham, says that some think them ill-advised but that the evidence does not point that way. Lord Rayleigh was able to obtain much from the published lectures, even if the students who listened to them were unable to interpret them.

Shortly after his return to London and the West End residence, Dr. Young married Miss Eliza Maxwell, the daughter of a cultivated and aristocratic family whose home was Calderwood Castle, Lanarkshire. She was a great help socially and assisted him in withstanding the waves of criticism and abuse which at times other scientists heaped upon him.

He continued his medical practice, but never was a popular or successful practitioner. His interests were too much bound up in science and scholarly pursuits to give him a popular bedside appeal. In 1810 he became one of the staff at St. Georges Hospital. Here, likewise, he attracted no acclaim for his medical ability but rather was recognized as a man of great talent as a philosopher.

This was the man of science who might easily have become a fashionable physician devoting his life to the treatment of the real and imaginary ailments of fashionable West End London society. Fortunately such a career was made unnecessary by Dr. Brocklesby's endowment of his scholarly talents.

Summary of the Period Preceding the Theory of Elasticity.—Turning our attention to France again, we find that we are entering upon a period of great activity and accomplishment in the fundamental growth of the Science of Elasticity. In 1809 the French Institute had offered a prize for "la théorie mathématique des vibrations des surfaces élastiques." There seemed to have been some difficulty in choosing the winner. The date for the first contest was October 1, 1811; a second was held October 1, 1813; and a third October 1, 1815. Mademoiselle Sophie Germain was a competitor in each of the contests and was finally judged the winner in 1815. Whether her sex had something to do with the indecision of the judges or the lack of agreement between theory and observation was the reason for three contests is not made clear. Mademoiselle Germain seems to have been the first woman to have achieved distinction in the field of applied mathematics. Her work was published at Paris in 1821 under the title *Recherches sur la théorie des surfaces élastiques*. Later investigations have shown that this paper of about 100 pages contained a considerable number of errata and incorrect assumptions.

From Galileo's beginning up to 1820 the net result of the labor of many workers was an inadequate theory of flexure; an erroneous solution of the tension problem; an unproven vibration theory for plates and bars; the differential equation for deflection of beams; Hooke's law; and Young's modulus of elasticity. This does not appear to be a great accomplishment for approximately 200 years of labor by some of the greatest mathematical minds of the world. But this labor, while it may not have produced specific accomplishments of great value, was laying the ground-work for later investigations. It had established the concept of normal forces at a beam section which was to grow into the theory of stress. The recognized distinction between slip or shear and extension or compression was to lead to the crea-

tion of the theory of strain. The differential equations for beams were to be further developed, and also the principle or virtual work from Mechanics was to serve its useful purpose. The science was now equipped with the tools which would permit able men to go on and to produce the solutions of the increasing number of problems of structural and mechanical design.

The Theory of Elasticity was about to be born. The task of this theory is to calculate the state of stress and strain in a solid body which is acted upon by a system of external forces or is in a state of slight internal relative motion, thereby obtaining information useful for engineering design.

The growth of the Science of Mechanics of Materials, as we have already observed, was not a straight-forward march but was beset with frequent retrogressions brought about by errors or the incorrect hypotheses of the mathematicians or experimentalists. In general, however, the direction has been continuously forward, even if the rate was slow and the path was devious.

During the first two centuries (1638–1820) of the growth of the Mechanics of Materials, even though each of the various investigators was interested in his own special problem, there was created a common interest in the physical constitution of materials with which they were working. Newton (*Optiks*, second edition, London, 1717) had presented the theory that material bodies were made up of small parts or molecules which acted on each other through a system of central forces. Newton believed these parts to be of finite size and definite shape. His followers reduced the size of the parts to that of material points or molecules. It was Navier, using Newton's conception, who was to be the first to work out the general equations for equilibrium and vibration of elastic solids.

CHAPTER 12

Theory of Elasticity

The First Testing Machine.—The decade from 1820 to 1830 may well be considered the most important period in the development of the basic principles of Elasticity. To three French scientists, Navier, Poisson, and Cauchy, Professors at the École Polytechnique in Paris, properly belongs the major credit for this accomplishment. Others, like Sophie Germain with her work on the normal vibrations of elastic plates and the able French physical elastician Savart (1791–1841) whose vibration experiments are spread through the pages of *Annales de Chimie et de Physique* from 1819 to 1840, also were helpful and suggestive.

In 1827 P. Lagerhjelm, a Swedish physicist, published the results of many experiments on density, elasticity, malleability, and the strength of cast iron and wrought iron. Lagerhjelm also appears to have been the first to employ a testing machine involving the use of the hydraulic press and the balanced lever. He probably should be credited with the invention of the mechanical testing machine. These and others were the able lieutenants during the early years of the development of the Theory of Elasticity.

Professor Pieter van Musschenbroek (1692–1761) of the University of Leyden is also credited[1] with the construction of a tensile machine prior to 1729. He used the principle of the steelyard. The machine was a crude affair with no means of keeping the steelyard level during the progress of the test.

The first machine of sizable capacity (18 tons) was built in 1768 by Jean Rodolphe Perronet (1708–1794). Next came

[1] "History of Testing Machines," by C. H. Gibbons, *Trans. Newcomen Society*, Vol. 15, 1934–1935.

Gauthey (1732–1806), Inspector General des Ponts et Chaussées, who built a beam-and-scale compression machine. He was followed by Jean Baptiste Rondelet (1743–1829), who introduced knife edges and a screw for strain compensation. His machine had a capacity of 100 tons. This was the machine which P. S. Girard (1765–1836) used for his experiments. The Brenton Cable and Chain Manufacturing Company seems to have built the first English machine.

The first testing machine to possess the features of the modern machine was designed by William Williams in 1829 at South Wales. It had both hydraulic power and dead weights for measuring the load. Its capacity was 130 tons. The Franklin Institute (1832–1837) possessed the first machine built in the United States.

The General Equations for Theory of Elasticity.—On May 14, 1821, M. Navier (1785–1836) read before the Paris Academy of Sciences a paper which was the first important general mathematical investigation of the theory of elastic solids. He had presented a paper on flexure of elastic plates on August 14, 1820, but this was considered only sufficiently important to print as an abstract. In the 1821 paper Navier presented the general equations of equilibrium and motion which must be true at every point in the interior, and also those for every point on the surface, of an elastic body. Navier's method of attack was to write equations expressing the equilibrium and vibrations of the molecules of a solid based on consideration of the laws of repulsion and attraction. Considerable opposition arose because, where triple summations were indicated, Navier substituted integrations which were not believed to be valid. The paper was finally published in Vol. VII of the Mémoires of the Paris Institute, 1827. The criticisms and discussions which arose may be found in Todhunter and Pearson's *History of the Theory of Elasticity*, published at Cambridge in 1886.

The First Textbook on the Mechanics of Engineering. In 1826 Navier published the first good tcxtbook on the mechanics of engineering. It was entitled, *Résumé des Leçons données à l'école des ponts et chaussées sur l'application de la Mécanique a l'établissement des constructions et des machines*. This text was revised in 1833 by Navier and again in 1864 by the great elastician, Barre de Saint-Venant, who added his own solution for torsion of non-circular sections. In the preface to the first edition Navier defines the function of structural analysis as the improvement of design. The book covers stress computation in beams of any cross-section by using Bernoulli's assumption of plane sections remaining plane after flexure; deflection of beams by the double-integration method; simple, curved, and continuous beams; torsion of circular sections; and two-hinged elastic arches. Navier also presented the idea that certain types of statically-indeterminate structures become statically determinate when the elastic conditions are taken into consideration. Navier thus seems to have been the first to solve an elastic statically-indeterminate structure.

Formulation of the Flexure Theory.—To Navier must be given the credit for the final formulation of the flexure formula for beams, and also for the division of the science into its two parts: (1) Theory of Elasticity and (2) Mechanics of Materials. However, in his first presentation of this widely used formula, Navier made the erroneous assumption that for any cross-section the moments of the compressive forces and the moments of the tensile forces were equal; this is true only for symmetrical sections. Navier's task was the consolidation of work of Galileo, Hooke, and Coulomb on flexure with Young's modulus of elasticity. Navier's equation contains no single letter representing moment of inertia, this quantity being presented in its original integral form. Navier did not call his equation a flexure formula but termed

it the "equation of permanent cohesion." His book also gives the solutions of a number of statically-indeterminate problems by taking into account the elasticity of the reacting bodies or supports. This method was first presented by Navier in his lectures in 1824 and published in the *Bulletin Philomatique* for 1825, page 35. Thus, to Navier belongs the credit for the first solution of an elastic statically-indeterminate structure. This pioneer text on structural analysis was an authoritative volume for almost a half-century.

Before producing his general equations of elasticity and writing his text on structural mechanics, Navier had written several other papers on the flexure of straight rods, curved rods, and slabs, but this work was not of much value. Previous to 1819, Navier seemed to be in error about the location of the neutral plane.

Poisson's Contribution to the Theory of Elasticity.—On August 1, 1814, after the first prize contest to which Mdlle. Sophie Germain had contributed, Poisson (1781–1840) gave *Mémoire sur les surfaces élastiques* to the French Institute. This apparently was Poisson's first paper before the Institute. It was followed during the next few years by several other papers, among which was *Note sur les vibrations des corps sonores* (*Annales de Chimie*, Vol. 36, 1827). In this paper he apparently claims to have determined the laws of normal vibrations but says that they are too complicated to present in the paper.

On April 14, 1828, Poisson read to the Paris Academy a *Mémoire sur l'équilibre et le mouvement des corps élastiques*, which was published in the Mémoires of the Academy for 1829. In the introduction of the paper Poisson gives a brief historical sketch of the Theory of Elasticity. Leibniz and the Bernoullis—James and John—had determined the equation of the catenary, for which Galileo had given an erroneous solution; James Bernoulli had investigated the

form of an elastic lamina when in equilibrium; D'Alembert had solved the problem of vibrating chords, and this solution was followed by another solution by Lagrange; and Euler and Daniel Bernoulli had examined elastic laminas, rods, or columns with various conditions of support at the ends.

Poisson then proceeded to establish the equations for the equilibrium of an elastic body—the three which hold for every point on the surface and the three which hold for every point of the interior. He says that these equations agree with those previously given by Navier, but no reference is made to any work by Cauchy or any other person. This fact seems to indicate that Navier should be given credit for first establishing these equations.

The publication of this paper by Poisson touched off another of those famous professional feuds. This one between Navier and Poisson reached the pages of several publications. Navier apparently felt that his work had not received adequate recognition from Poisson. He claimed to have done the pioneering work for the correct discussion of elasticity and thought that he should have been given more consideration—and this now is evidently the viewpoint of others.

Navier credited Mdlle. Germain with the creation of a true and ingenious principle in her work on elastic surfaces, but Poisson disagreed and claimed that her idea was not sound. Navier defended Mdlle. Germain's hypothesis. The controversy between Poisson and Navier continued for some time. The later discussion of the subject seems to have decided many points in Poisson's favor, but he might well have been somewhat more liberal in his recognition of Navier's pioneer position in the field, even though his own contributions are now generally considered more important and valuable. Poisson's *Mémoire* also brought out the now well known Poisson ratio, the relation between the deformation perpendicular to the line of action of the stress and the deformation parallel to the direction of the stress.

Origin of the Theory of Stress.—The Paris Academy appointed a commission to examine Navier's paper of August 14, 1820. One of these men was Cauchy (1789–1857), whose interest was so stimulated that in two years, on September 30, 1822, he presented a paper entitled *Recherches sur l'équilibre et le mouvement intérieur des corps solides ou fluides élastiques ou non-élastiques*. This paper appeared in *Bulletin Philomatique* of 1823, and covered practically all the important elements of a complete elementary theory of elasticity for an elastic body. Without use of any mathematical symbols he states the six stress-components which must be considered at any point in a body, and also presents a number of other concepts which only a person who thoroughly understood the problem could have developed. He had conceived the idea of stress at a point, the principal planes of stress, and axes of principal strain. The paper is historically important because it contains the origin of the theory of stress. It presents what has been called Cauchy's Theorem. The stress on any infinitesimal face in the interior of a solid or fluid body at rest is the resultant of the stresses on the three projections of this face on planes through its center. This paper was the first of many on elastic problems which, under various titles, flowed from the pen of this capable mathematician during the three decades to follow.

In his later work, *Exercices de mathématiques* (2nd Vol., 1827), Cauchy's general equations expressed the elastic behavior of a body by two constants. This seems to be the first use of two constants in the equations of isotropic elasticity. Navier used one constant in his isotropic elasticity, which Cauchy considers a very special case of bi-constant isotropy.

Controversy over the Number of Elastic Constants.—The fundamental theory and concepts of the Theory of Elasticity now having been established by Navier, Poisson, and

Cauchy, the technique of applying these principles to practical problems had to be developed. Many applications to statically elastic and vibration problems had to be made before this new basic theory could be satisfactorily tested and proved.

As soon as the principles of elasticity began to be better understood, the question of the number of elastic constants required in the equations became a troublesome one. Navier's original equations contained one constant, while Cauchy used two constants. For elastic isotropy were there to be one constant or two constants, and for elastic aeolotropy were there to be fifteen or twenty-one? The first alternative became known as the rari-constant theory, and the second as the multi-constant theory. Rari-constant advocates numbered in their group some highly influential names—Navier, Poisson, Clausius, Grashof, and even Saint-Venant. Cauchy is also sometimes mentioned, but he was the first to introduce multi-constancy.

It appears that the real difficulty was caused by the inability to obtain isotropic material; even the metals which the rari-constant elasticians maintained were isotropic and therefore suitable to their rari-constant theory could not be considered isotropic when they had been worked (rolled or drawn). It later was recognized that it was difficult to decide whether a metal plate or rod should be treated as isotropic with two constants or aeolotropic with the rari-constant theory applied.

In 1852 M. G. Lamé published a book of 335 pages entitled *Leçons sur la théorie mathématique de l'élasticité des corps solides*. It contains 24 lectures and is one of the few books written by an eminently able man which is sufficiently elementary to be easily readable. It is an exceedingly worth-while contribution to the literature of the subject. In the fourth lecture Lamé shows that for a homogeneous body of constant elasticity in all directions the coefficients of

elasticity can be reduced to two. This book seemed to bring about the general swing toward the multi-constant method. Even Saint-Venant, who had been one of the proponents of rari-constancy, used the multi-constant theory in his later work on torsion and flexure. The publication in England a few years later (1867) of Thomson and Tait's *Natural Philosophy* seems to have finally settled the controversy in favor of multi-constancy. The fifth lecture introduces the ellipsoid of elasticity, or what is called Lamé's stress-ellipsoid, which gives the magnitude of the stress in any direction around a point; and also introduces the surface of the second degree which gives the direction of the stress across any element.

While—or possibly a bit before—the physical properties of iron were being investigated in England, Lamé and Clapeyron seem to have been doing much the same thing in Russia. A description of their work may be found in *Annales des Minas* (Paris, 1825). Here also is mentioned a testing machine invented by Lamé and constructed in St. Petersburg. It is a variation of the hydraulic testing machine built by Lagerhjelm.

Clapeyron's Three-Moment Theorem.—In *Comptes rendus* (Paris, 1857), Tome XLV, page 1076, there is a digest of a paper by E. Clapeyron in which Clapeyron's Theorem, or the three-moment theorem, for uniform loads is discussed. Both Navier (1825) and Berlanger had previously attempted to solve this problem, but their final equations contained terms involving the beam reactions. Clapeyron's Theorem was later expanded by J. M. Heppel (*Proceedings of the Institution of Civil Engineers*, Vol. XIX, London, 1859, W.) to include concentrated loads. There now seems to be some doubt about the validity of Clapeyron's authorship of the theorem. It is supposed to have been developed in 1855 by one Bertot.

Development of Mechanics of Materials in France.—As we have previously stated, when a truly great man appears, and especially if he is a great leader or teacher, he is usually associated with or followed by others who demonstrate notable ability. In France in connection with the development of Mechanics of Materials, it was Coulomb (1736–1806) who was followed by Navier (1785–1836), Poisson (1781–1840), Cauchy (1789–1857), the very capable and prolific Barre Saint-Venant (1797–1886), and other minor contributors. This was indeed a productive period for France in mathematical science. The French were very much in the lead. The other countries of Europe had produced nothing of importance for some time, and the western hemisphere apparently was too busy with its geographical expansion and commercial growth to develop colleges or technical schools with staffs capable of carrying on research in the advanced mathematical sciences. However, as we turn to England and Germany, we discover evidence of progress along somewhat more practical lines.

In *Kunst und Gewerbe-Blatt des polytechnischen Vereins für das Königreich Bayern* (Munich, 1830), we find a paper which describes another testing machine. This machine, however, does not seem to have been superior to that which had been invented by Lagerhjelm.

The Basic Principle of Photo-Elasticity.—In Brewster's edition of *Ferguson's Lectures on Select Subjects* (3rd Edition, 1823, Volume II, page 232), there appears a description of an apparatus invented by Brewster for investigating strains in glass models. Brewster described the basic phenomenon of photo-elasticity—double refraction caused by strain—before the Royal Society on Feb. 19, 1816. The apparatus consisted of a source of polarized light, means of subjecting the models to measurable loads, a screen, and a lens for projection of an image. This was the beginning of what in more

recent years has become one of the most useful techniques for stress analysis—the photo-elastic method of stress analysis. Brewster called his invention the Teinometer.

No important practical application was made of Brewster's discovery until Carus Wilson applied it to the study of the stresses in a beam caused by a concentrated load (*Philosophical Magazine*, Vol. 32, 1891, page 481). Ten years later, in studying the stress distribution of arch bridges, A. Mesnager also used the device (*Annales des ponts et chaussées*, 4 Trimestre, 1901, page 129).

The English Experimentalists Contribute.—In England there is little in the nature of theoretical growth during the first half of the nineteenth century. Considerable experimental work was conducted on the determination of the physical strength of cast iron and some other materials such as wood, bone, and marble. Eaton Hodgkinson apparently was the most competent and prolific experimentalist in England for the decade from 1830 to 40. Work was also done by Peter Barlow, Thomas Tredgold, W. Fairbairn, and Thomas Thomson. Hodgkinson's experiments dealing with impact on beams and the testing of small cast-iron columns seem to have been the most important work done. The net result, however, was no advancement of fundamental theory.

In 1843 H. Moseley published *The Mechanical Principles of Engineering and Architecture* (London). This book contains a chapter on Strength of Materials and another on impact, with some problems on resilience. Moseley was responsible for the introduction of the work of the great French elasticians into England.

Sir William Fairbairn appears to be the first to make a comprehensive study of riveted joints (*Philosophical Transactions*, Part II, 1850, page 677). The experiments described in this paper were made in 1838–39, but were not presented until the later date because of Fairbairn's other professional

duties. It appears that Eaton Hodgkinson did much of the experimental work for Fairbairn. The purpose of these experiments was to determine the suitability of iron plates and riveted joints for ship construction. For many years the results of these experiments constituted the only experimental data available on riveted joints.

James Clerk Maxwell (1831–1879), Professor of Experimental Physics at the University of Cambridge, wrote a paper which was published in the Transactions of the Royal Society of Edinburgh for 1850 under the title of *On the Equilibrium of Elastic Solids.* Maxwell claims that the mathematical theory of elasticity presented by Navier, Poisson, Lamé, and Clapeyron is not consistent with the results obtained by experiments. He maintains that formulas involving one coefficient are insufficient to represent actual conditions. This was a renewal or a continuation of the French discussions of rari-constancy and multi-constancy brought into English science. Maxwell proposed two axioms for the foundation of his theory.

If three pressures along three rectangular axes are applied to a point in an elastic solid, the following conditions exist:

(1) The sum of the three pressures is proportional to the sum of the compressions which they produce.

(2) The difference between two of the pressures is proportional to the difference between the compressions which they produce.

The equations which are derived from these axioms contain two coefficients, and they differ from those presented by Navier because they do not assume an invariable ratio between the linear elasticity and the cubical elasticity.

In this paper Maxwell also calculates what he calls the "optical effect of the pressure of any point." This terminology apparently indicates that he had not heard of *Die Gesetze der Doppelbrechung des Lichts in comprimirten oder un-*

gleichfömig erwärmten un Krystallinischen Körpern, published in Berlin in 1841 by F. E. Neumann who, using Brewster's work as a base, had built up a theory for the photo-elastic analysis of strains. He had shown that the optical axes of elasticity will coincide in direction with the axes of principal strains.

A few years later, in 1864, Maxwell introduced into the science of graphical statics what is now known as the Maxwell-Cremona diagram. This is the idea of linking together the force polygons for the various joints of a truss instead of drawing a separate polygon for each joint. Also in 1864 Maxwell wrote a paper on the *Calculation of the Equilibrium and Stiffness of Frames*, which was published in the *Philosophical Magazine*. In this paper the variables in the equations for the geometrical coherence of the structure are the redundant stresses. These equations are now known as the Maxwell-Mohr equations because they were also derived a few years later by Otto Mohr of Germany, who employed the principle of virtual work. However, Mohr's work was not published until many years later, in *Abhandlungen aus dem Gebiete der technischen Mechanik* (Berlin, 1914).

England was fortunate at this time since another great teacher and worker, Professor W. J. M. Rankine (1820–72) of the University of Glasgow, was just beginning to become prominent. In 1858 he published his *Manual of Applied Mechanics*, which was one of the first books to attempt to present to engineering students the results obtained by the use of the theory of elasticity. It was a classic in the field, having passed through over twenty editions. In the first edition (page 338) will be found the method still used for determining the shearing stresses in beams. There also is a method for the solution of earth pressures against a retaining wall which gives results that agree with those obtained by Coulomb's solution; and the now well known Rankine-Gordon formula for columns, for which Rankine gives credit

to Gordon. Rankine's *Applied Mechanics* was a distinct improvement in the English textbook standard.

Professor Rankine was a prodigious worker who wrote many papers on various problems. He has been followed in more recent years by Arthur Morley, the author of a widely read text *Strength of Materials* and another called *Theory of Structures*; and by A. E. H. Love, who is probably better known for his *Theory of Elasticity* (first edition in 1892–1893 and third edition in 1920). These men, with their practical predecessors and such experimentalists as Hodgkinson and others, are chiefly responsible for the English contributions to the early growth of what is now known as the Science of Mechanics of Materials.

Elastic Limit Defined.—Returning again to France, we find that G. Wertheim, a physical elastician of considerable ability, presented his first paper to the Paris Académie on July 18, 1842. This was entitled *Recherereches sur l'élasticité,* and it was followed by two other papers on the same subject. He was the first to define the elastic limit—as it is now defined—as the stress which produces a permanent set, and not as the stress at which the strain ceases to be proportional to the stress or the proportional elastic limit.

In a paper read before the Paris Académie on February 10, 1848, and printed in *Annales de Chimie* at Paris in 1848 as *Mémoire sur l'équilibre des corps solides homogènes,* G. Wertheim states that Cauchy has obtained the equations of equilibrium in a more general form than either Navier or Poisson and that the Navier-Poisson equations are a particular case of Cauchy's bi-constant form.

First Application of the Theory of Elasticity; Torsion of a Rectangular Prism.—Since Galileo's first experiments, many attempts had been made to solve the problems of flexure and torsion of bars without any practical theory which was definitely satisfactory having been discovered. Cauchy, in

his book *Exercices de mathématiques* (1827), was the first to apply the general equations of elasticity to practical problems. His work on the torsion of a rectangular prism was incorrect and had little value as a practical solution. It is, however, of interest historically. The books written for the practical men of the time used the flexural and torsional theory introduced by Coulomb. The Science of Elasticity had to wait for the arrival of that great mathematician Saint-Venant (1797–1886) before it could be brought down to the level where it could become a tool for the practical engineer rather than an interesting study for the super-mathematician. Saint-Venant made his first contribution to the Science of Elasticity in 1837 while serving as a deputy for Coriolis at *École des ponts et chaussées*. He published some sheets he called *Leçons de mécanique appliquée faites par intérim par M. de St.-Venant, Ingénieur des ponts et chaussées.*

Saint-Venant was the first to apply the general equations of elasticity successfully to the practical type of problem; to make the Theory of Elasticity of practical value was to be his life work. He early realized the difficulties involved in attempting to develop general solutions for the kinds of problems which the practical engineer desired to solve. Because it was impossible to control all conditions, he studied the methods of the old approximate solutions and from these he obtained suggestions which permitted him to simplify the general equations and obtain more easily usable relationships.

In the *Mémoires des Savants étrangers* for 1855, which was presented to the Academy of Paris on June 13, 1853, we find his results for the torsion of a prism. His assumption was that the state of strain was a simple torsion about the longitudinal axis combined with the variable longitudinal strain over its cross-section, and that plane sections would become curved surfaces after a torque was applied. The paper is mostly about the torsion of prisms, but a number of other

types of cross-sections were considered. The strains were small, with the stress well within the elastic limit. This was the most important fundamental theoretical development since Poisson's work on the basic equations of the Theory of Elasticity.

Theory of Elasticity Applied to the Flexure Problem. In 1856 the *Journal de Mathématique de Liouville* printed Saint-Venant's classical *Mémoire sur la flexion des prismes*, a paper of about 100 pages. Portions of this paper had appeared in *Comptes rendus* for 1854 and 1855. The first part is devoted to a historical survey of the accomplishments of his predecessors. He then proceeds to point out why the Bernoulli-Euler theory of flexure is incorrect. Saint-Venant's work on beams is involved and not of much value to those of average mathematical training.

Saint-Venant's theories of torsion and bending will be found in some books; others give a bending theory developed by Jouraveski in *Annales des ponts et chaussées* (1856) and Rankine in *Applied Mechanics* (first edition, 1858) and further improved by Grashof in *Elastizität und Festigkeit* (second edition, Berlin, 1878). Grashof also gives Saint-Venant's theory of flexure.

In an article *Sur la résistance des solides* which appeared in *L'Institute*, Vol. 24 (1856), Saint-Venant gives some interesting formulas for the moment of inertia and the location of the principal axes of plane figures.

"Résumé des Leçons" Revised by Saint-Venant.—In 1804 there was printed in Paris a most remarkable book of some 1100 pages entitled *Résumé des Leçons sur l'application de la mécanique à l'établissement des constructions et machines*. Navier was the author of the first part and Saint-Venant had added copious notes, so that the book became a complete textbook of elasticity from the practical standpoint. The material which had been added to Navier's original work

by Saint-Venant made the volume usable by the engineer. In his discussion of the accomplishments of his predecessors, Saint-Venant exhibited considerable ability as a historian by showing the interdependence of the successive advances made during the growth of the science through the years. This book of Navier's, as revised and expanded by Saint-Venant, is one of the historically important books of the Sciences of Strength of Materials and Theory of Elasticity.

Even though his writing and accomplishments were prodigious and important, the able band of disciples and students who gathered around or followed the leadership of Saint-Venant must not be lightly cast aside. Saint-Venant was the coordinator or liaison man between the founders of Theoretical Elasticity and its more modern and usable descendant, Mechanics of Materials. If Saint-Venant's work had been limited to the establishment of a correct theory of flexure and the theory of torsion which takes into consideration the distortion of plane sections, he still would have been one of the most influential contributors to the Science. He possessed both the theoretical mathematical ability and the required understanding, along with a knowledge of the practical requirements of the engineer. With Saint-Venant's contribution to their growth, the Science of the Theory of Elasticity and its more work-a-day half-brother the Science of Strength of Materials or Mechanics of Materials at last attained the dignity of maturity.

The Germans Contribute.—Over the years during which the French had been busy with the development of Mechanics, Mechanics of Materials, and the Theory of Elasticity, Germany had contributed little of value to this work. Evidently the German mind of this period was not well adapted to original fundamental analysis along mathematical lines. We find few German names in the early history of our sciences, and none are of major importance.

J. Weisbach's *Ingenieur-Mechanik*, published in 1848 and followed by revisions in 1850, 1856, and 1863, appears to be the first important German book. It was later translated into Polish, Swedish, Russian, and English, and also was published in an American edition. This book was a great improvement over the earlier texts, even though it contains some erroneous material. It became the standard German text for many years but was later largely replaced by Grashof's *Elastizität und Festigkeit*, published in 1878.

In 1858 E. Winkler published *Formänderung und Festigkeit gekrümmter Körper, insbesondere der Ringe* in *Der Civilingenieur*, Bd. IV, S. 232, Freiberg. This was an important paper theoretically and practically, even though it contains many errata. It was the first satisfactory attempt to determine a theory for the strength of chain links, which was a much too complicated problem for investigation by the Theory of Elasticity. However, Saint-Venant's work on flexure had shown that the Bernoulli-Eulerian theory gave exceedingly accurate results for straight bars. Further discussion of this problem will be found in *Elastizität und Festigkeit* by F. Grashof (1878), page 251, and *A History of Elasticity and Strength of Materials* by Todhunter and Pearson (1893), Vol. II, Part 1, page 422. The exact solution was finally published in 1881 by H. Golovin in *Bulletin of the Institute of Technology* at St. Petersburg; by C. Ribiere in *Comptes rendus*, Vol. 108 (1889) and Vol. 132 (1901), in France; and by L. Prandtl in a paper by A. Timpe in *Zeitschr. f. Math. u. Physics*, Vol. 52 (1905), page 348.

The next historically important paper contributed by the Germans was an attempt by C. Neumann to obtain the fundamental equations of elasticity in a new manner. His work will be found in the *Journal für Mathematik*, Vol. 57 (1860), S. 281, Berlin, under the title *Zur Theorie der Elastizität*. This discussion was based on uni-constant isotropy and therefore has little practical application since most mate-

rials are aeolotropic; however, the mathematical work is good.

Also W. Luders in *Dingler's Polytechnisches Journal,* Bd. 155 (1860), S. 18, Stuttgart, and in *Polytechnisches Centralblatt Jahrgang* for 1860, Cols. 950, Leipzig, first called attention to what are now known as Luders lines.

We now come to the great teacher and leader of the German physicists, Franz Neumann, who taught at Königsberg and lectured on elasticity from 1857 to 1874. His lectures were published in 1885 under the editorship of O. E. Meyer with the title *Vorlesungen über die Theorie der Elastizitä tder festen Körper und des Lichtäthers,* a volumê of some 374 pages from the notes of four of Neumann's students. Franz Neumann was the real founder of the German interest in elasticity as later carried forward by Kirchhoff, Clebsch, and others. K. Pearson says "Neumann's lectures form the best elementary treatise on elasticity and its relation to light that I have met with in the German tongue."

Considerable work by Kirchhoff appears in various German journals, especially reprints in *Gesammelte Abhandlungen,* of which he was the editor (1882). Kirchhoff's papers are largely attempts to improve on previously written material, and the results are often far from satisfactory, either in clarity or in accuracy. His work on the equilibrium and motion of elastic wires constitutes the portion of his writing which is of most value to those interested in Structural Mechanics. Thomson and Tait in their *Natural Philosophy,* Part 2 (page 609), refer to this part of Kirchhoff's work as "the first thoroughly general investigation of the equations of equilibrium and motion of an elastic wire."

A. Clebsch, one of Germany's greatest mathematical geniuses, was born in 1833. His first memoir was published in 1860 in *Crelles Journal für die reine u. angewandt Mathematik* when Clebsch was twenty-seven. Two years later (1862) he published a 424-page book entitled *Theorie der*

Elastizität du fester Körper. The preface states that the object of the book is to establish a sound basis for practical studies and applications. A brief examination of the text will quickly convince the reader that Dr. Clebsch did not attain his objective. Clebsch's book is something which is suitable for the mathematical elastician but not for the practical engineer. Most of his work refers to isotropic materials. The book has been translated into French by Saint-Venant and Flament. The notes and corrections made during the translation make the French edition a much better book. The book has slight value for technical students. It is, however, an exceptional example of how a great mathematical genius can become so interested in original investigation that he loses all sense of the relative practical value of his problem. It is the old story of a genius being unable to write a book which is understandable to any but his peers. Clebsch died in 1872 when only thirty-nine years of age. It is difficult to imagine what sort of book this 29-year-old brain would have produced if it had been permitted to live thirty or forty years longer instead of only ten. However, the book is not without value to those who are properly equipped to follow the analytical methods of the author. The mathematician will appreciate the ability of the author and will be stimulated and intrigued by the suggestive character of his solution.

CHAPTER 13

Post-Saint-Venant Period

We have now reached the point in the analytical growth of Mechanics of Materials and the Theory of Elasticity where the basic fundamental principles seem to have been fairly completely established. Looking back to the pioneer efforts of Galileo, we find that this particular branch of science is almost entirely indebted to the able French mathematicians and physicists for the creative work, with some small assistance from the Italians and English in the early period and later from the Germans. America and the western hemisphere contributed nothing, probably being too busy with the vital problems of living to devote either time or energy to speculative creative effort. Up to this period, the latter part of the nineteenth century, the growth of the Science had to a large extent taken place behind national boundaries. Methods of communication and cooperation across these national frontiers were not too effective, nor were they often desired. The result was more or less independent development, with the French so far out in front that the rest of the world seemed to be just hitch-hikers waiting beside the highway of knowledge for a bit of a lift.

With the invention of improved communication and transportation facilities, these national and natural barriers to cooperative research began rapidly to break down, even though there were no important changes in the political relationships. The common interest of all peoples in the improved living conditions resulting from the growth of science probably had much to do with the lowering of barriers so that, from this period on, we may at least begin to think of the growth and perfection of science as a problem which is a world-wide project with many nations par-

ticipating and not as a problem which three or four secluded workers are trying to solve behind closed doors.

The demands of industry and engineering were becoming so insistent that the professional-student type of scientist, who secluded himself in cloistered halls and devoted his entire time and energy to the development of mathematical theory and natural philosophy unconnected with the practical problems arising from the application of science to industry, was beginning to be less influential. His place was eventually to be taken by the more aggressive, daring and utilitarian, scientifically trained engineer, who was generally a man of broad practical vision coupled with sound theoretical training.

Squire Whipple's Truss Solution.—When we review our American publications the first contribution of value from the United States appears to be Squire Whipple's *Elementary and Practical Treatise on Bridge Building*, published in 1847 (2nd edition, 1873) by the inventor of the Whipple truss. In this book Whipple, who later was made an honorary member of the American Society of Civil Engineers, gives us the first solution of a jointed truss by using the horizontal and vertical components of the forces and simple statics. Four years later (1851) Herman Haupt, also an American, produced another method which was published in *General Theory of Bridge Construction* for 1869 but which apparently was not so good as Whipple's work. It was not until 1865, when August Ritter published *Lehrbuch der Technischen Mechanik* containing his now much used method of sections for computation of truss stresses, that a really satisfactory method was available for such calculations.

Graphical Statics Revived.—At this period that exceedingly useful method of solution now generally known as graphical statics was beginning to come into use again in many places. It had been the original tool of Mechanics, as

Newton's *Principia* and the writings of other early authors clearly demonstrate; but, as analytical methods became more effective, it was largely superseded. However, when Professor C. Culmann (1821–1881) of the Polytechnikum at Zürich, Switzerland, published his *Graphische Statik* in 1864–66, new life seemed to be generated. *Graphische Statik* contains stress solutions by the Ritter method of sections by graphically obtaining the components of all known forces on each free body parallel to the unknown stresses. The book also contains some work with the string polygon. Culmann's method was improved on by Clerk Maxwell (1831–1879) of Cambridge University who in the same year (1864) invented the Maxwell-Cremona stress diagram, which has been mentioned. This diagram will be found in the *Philosophical Magazine* (4), Vol. 27 (1864), page 250. It was followed in 1867 by the influence line which was the work of E. Winkler, professor at the Zürich Polytechnikum. In the period from 1885 to 1906 W. Ritter, also of the Zürich Polytechnikum, published a four-volume work called *Anwendungen der graphischen Statik nach Professor Dr. C. Culmann.*

In 1868 Professor Otto Mohr (1835–1918) of the Technische Hochschule at Dresden, Germany, developed a graphical method for beam-deflection problems by using the string polygon. To Otto Mohr must also be credited the well known Mohr diagram (*Zivilingenieur*, 1882, page 113) for stress analysis at a point in a plane and three-dimensional stress at a point, as well as several graphical devices which he applied to the solution of continuous beams. Mohr seemed to have an unusual proficiency in the graphical technique.

Area-Moment Method Discovered.—In 1873 Dean Charles E. Greene taught his classes at the University of Michigàn for the first time his area-moment method of computing beam deflections. Five years earlier, in 1868, Mohr had developed an analytical method which was somewhat

similar to Dean Greene's method; however, Mohr's work was not published until 1906 in *Abhandlungen aus dem Gebiete der technischen Mechanik* at Berlin. It did, however, form the basis for a general principle evolved by H. Müller-Breslau in 1885 and published in *Beitrag zur Theorie des Fachwerks, Zeitschrift des Architekten und Ingenieur-Vereines zu Hannover.*

Maxwell Solves the Statically-Indeterminate Structure. Six years before Clerk Maxwell had published in England his solution for statically-indeterminate structures (1864), there appeared in *Comptes rendus,* Vol. 46 (1858), Paris Academy, another method for the solution of statically-indeterminate structures by an Italian, L. F. Menabrea; he, however, did not satisfactorily develop the proof for his solution. In 1875 A. Castigliano (1847–1884) published *Toria Intorno dell Equilibrio dei Sisterni Elastici,* in which he successfully established the method of least work which is now known as Castigliano's Theorem. Castigliano's work may also be found in his book *Théorie de l'équilibre des systèmes élastiques* published at Torino in 1879, or in an English translation *Elastic Stresses in Structures* by E. S. Andrews, published at London in 1919.

H. Müller-Breslau (1851–1925), Professor at Technische Hochschule at Berlin, in his books *Die neueren Methoden der Festigkeitslehre* published at Leipzig in 1886 and *Die Graphische Statik der Baukonstruktionen* published at Leipzig in 1892, presents a third method and illustrates its application to solid frames and trusses when settling of the supports is taken into account. This method is a variation of the Maxwell method, the redundant stresses being used as the variables. The geometrical coherence is expressed by linear equations obtained from superposition of the displacements produced by the separate loads on the structure. The displacements caused by unit loads are the coefficients in these equations.

Because of the general form of the Müller-Breslau equations, any method of displacement computation may be used for determining the values for the various terms. *Die Deformations methode* by A. Ostenfeld of Denmark in 1925 gives a fourth method of solution of statically-indeterminate structures; this method uses the deformations as the unknowns. The equations are of the same general form as the Maxwell-Mohr equations, but the loads and the deformations are interchanged.

Professor Maxwell of Cambridge University is the successful pioneer in statically-indeterminate structures, even though Menabrea preceded him by six years in attempting a solution. All the other workers in this field seem to have drawn their inspiration from Maxwell's work.

The French Go Into Eclipse.—With the passing of Saint-Venant the French seem to have gone into eclipse. The establishment of the fundamental equations of elasticity and Saint-Venant's contribution of the applications of this theory to the torsion and flexure problems made the science ready for the work of those who were to proceed with its application to practical problems.

The task now was to perfect the operational techniques and to apply these techniques to the ever increasing number of problems arising from the practical world where the successful development of large-size power-producing units, the establishment of power-driven transportation with the accompanying advances in machine tools, and the beginning of scientific metallurgy were making themselves felt in a new mode of life.

Up to this time it had been almost entirely a muscle world whose strength demands on the materials of construction were generally of a moderate nature; but with the creation of steam and electrical power it became ever more important that man be able to predict the behavior of his creations

before they were built and not have to depend on actual trial to learn if his designs were successful.

The Germans Make Practical Applications of the Theory of Elasticity.—It was at this point that the Germans with some help from the English began to apply the established principles to practical problems. A. Föppl (1854–1924), Professor at the Technische Hochschule at Munich, and his son L. Föppl, who later succeeded to his father's professorship, were the German pioneers in the application of the Theory of Elasticity.

In 1907 A. Föppl published at Leipzig a book on the Theory of Elasticity which was the last volume of his five-volume *Technische Mechanik*. Later, in 1920, A. Föppl and his son L. Föppl collaborated on a two-volume theory of elasticity for engineers entitled *Drang und Zwang*. In these books the Föppls make use of the principle of minimum energy for variation of shape which W. Ritz first published in 1908 in *Crelles Journal* and also in 1909 in *Annales Physik*, Vol. 28.

In addition to his authoritative books the energetic senior Föppl is credited with the authorship of eighty-five technical and scientific papers, the titles of which were published in *Zeitschrift für Angewandte Mathematik und Mechanik* of 1924—a production capacity attained by few men.

A. Föppl and L. Prandtl did notable work on torsional problems. Prandtl also discovered the soap-film method for determining torsional stress. He demonstrated that the equations for the state of stress of a twisted bar are of the same form and have the same boundary conditions as the equations for the surface of an elastic membrane stretched over an opening of the same shape as the cross-section of the bar and distended by subjecting the membrane to a pressure differential on the two sides (*Physik Zeitschrift*, 1903, page 708; also *Jahresberichte d. Deutsch Mathematik Ver.*, Vol. 13,

1904). Prandtl also did pioneer work on the theory of curved bars.

L. Föppl, the son, published a noteworthy article, *Neuere Fortschritte der Technischen Elastizitätstheorie*, in *Zeitschrift für Angewandte Mathematik und Mechanik*, 1921, Vol. 1, in which he discusses the theory of elasticity from the standpoint of its more modern applications.

Another notable German writer is H. Reissner, whose contributions to the theory of buckling of beams and thin plates, bending of thin plates, and buckling of uniformly compressed rectangular plates appear in a number of papers. Also there were Dr. A. Nadai and Dr. Th. von Karman. These three men have since become residents of the United States.

While at Göttingen, Dr. Th. von Karman did considerable investigating of problems involving stresses above the yield point, especially in connection with columns. One of his papers was published in 1910 in *Forschungsarbeiten*, Berlin; a paper on the bending of curved tubes was published in *Vereines Deutscher Ingenieure*, Vol. 55 (1911); and important papers on bending of thin plates were published in *Encyklopädie der Mathematischen Wissenschaften*, Vol. IV (1910), and in *Trans. A.S.M.E.*, Vol. 54 (1932). Since coming to the United States, Dr. von Karman has been director of the Guggenheim Aeronautical Laboratory at the California Institute of Technology. A large amount of important research in the fields of aeronautical engineering and fluid mechanics has been carried on at the Guggenheim Laboratory under Dr. von Karman's direction. In 1941 some of Dr. von Karman's friends published a Sixtieth Anniversary Volume containing a biographical sketch and a bibliography of his contributions to the technical literature of the world. The book also contains a number of interesting papers by friends and students of Dr. von Karman.

American Engineers Become Theory Conscious.—In the United States the American Engineering Societies have now been in existence for more than a half-century. When we examine the proceedings for the early years of their existence, we are at once impressed by the paucity of the mathematical type of paper. There appears to be little of an important mathematical nature until we reach the second decade of the twentieth century, and even then the number is scanty until after 1925. Apparently the engineer of the early years of these societies was generally a quite different type of man from the engineer of the present era. This can, no doubt, be accounted for by the nature of his work which was more or less pioneering and not the more exacting refined design that is the product of our present-day high-speed requirements and great structures.

In *Trans. American Society of Civil Engineers,* Vol. 75 (1912), we find a paper on *Faults of the Theory of Flexure,* by Henry S. Prichard, which devotes some 100 pages, including the discussion, to the flexure theory. This paper should be of special interest to students for its historical value.

In *Trans. American Society of Civil Engineers,* Vol. 76 (1913), there is a paper on the *Strength of Columns,* by W. E. Lilly, which is also of interest historically. It presents nothing new but does show that columns were just as controversial a subject to the engineers of that day as they are today. Progress in this field does not seem to have been sensational. The paper also contains a good bibliography of column literature up to publication.

Photo-Elasticity Becomes a Practical Tool.—The year 1920 saw the beginning of the modern interest in the technique of photo-elasticity as a successful stress-analysis tool. The discovery of the basic principle used in photo-elastic stress analysis, the double-refraction property of certain materials when subjected to strain, was first presented by

David Brewster in a paper in the *Philosophical Transactions* for 1816. Brewster's work was qualitative, not quantitative. Later the results of additional experiments were given by M. Biot in *Sur une nouvelle propriété physique qu' acquirèrent les lames de verre quand elles exécutent des vibrations longitudinales*, which was published in *Annales de Chimie et de Physique*, Vol. XIII (1820), and by A. Fresnel in *Résumé d'un Mémoire sur la réflexion de la lumière*, published in *Annales de Chimie et de Physique*, Vol. XV (1820). However, the first systematic investigation of the complete theory of the stress-optical effect was made by F. E. Neumann in *Die Gesetze der Doppelbrechung des Lichts imconprimierten oder ungleichförmig erwärmten un Krystallinischen Körpern*, published in *Abh. d. Kon. Acad. d. Wissenschaften zu Berlin* in 1841. Eleven years later Clerk Maxwell, apparently unaware of Neumann's work, also discovered the stress-optical laws and published *On the Equilibrium of Elastic Solids* in *Trans. Royal Society Edinburgh*, Vol. 20 (1853). The next important work on photo-elasticity was by G. Wertheim and is described in *Mémoire sur la double réfraction temporairement produite dans les corps isotropes, et sur la relation entre l'élasticité mécanique et élasticité optique*, published in *Annales de Chimie et Physique*, Ser. III, Vol. XL (1854).

Brewster had suggested that photo-elasticity might be applied to the determination of stresses in structures; but Carus Wilson, *Philosophical Magazine*, Vol. 32 (1891), was the first engineer to make actual use of the method. He used it in examining the stresses in a beam carrying a concentrated load; and further applications were made by A. Mesnager, *Annales des Ponts et Chaussées*, 4 Trimestra (1901), p. 129, and 9e Series, Vol. 16 (1913), p. 135.

However, the real development work in the technique of photo-elastic investigation is due to Professor E. G. Coker who introduced models other than glass, using bakelite, celluloid, etc. See *General Electric Review*, Vol. 23 (1920),

page 870, and *Journal of Franklin Institute*, Vol. 199 (1925), page 289. In 1931 Professor Coker and Professor Filon published their book *Photo-Elasticity*, which contains a sixteen-page bibliography. It was followed in 1935 by a much smaller volume *Festigkeitslehre mittles Spannungsoptik*, by L. Föppl and H. Neuber, Berlin.

American Engineers Introduced to the Theory of Elastic Structures.—The next paper of note in the *Trans. American Society of Civil Engineers* is in Vol. 85 (1922). It is *Buckling of Elastic Structures*, by H. M. Westergaard, then Assistant Professor of Theoretical and Applied Mechanics at the University of Illinois. This is the first important theoretical discussion by an American engineer of the buckling of such elastic structures as columns, curved members, and slabs, and includes a four-page classified bibliography. The nature of the paper and the reception which it received can best be conveyed by the following quotations from the printed discussion: "Unfortunately by a mathematical physicist . . . the language he speaks is foreign to most of his readers," and "The paper is probably the most thorough treatise that the writer has seen on the theory of combined buckling and cross-bending of columns and slabs," and "Only a few men in the profession outside the colleges will be able to follow this paper or even understand and use the formulas derived." Thus, did Professor Westergaard set the American engineering profession back on its heels in 1922. The paper is something which every modern engineer who wishes to be well informed on such theory should read. The bibliography also is exceedingly valuable. This paper is a good forecast of what was ahead for the American engineering profession. The American engineer was now going to need a sound mathematical and theoretical training if he was to have a prominent part in enhancing the prestige of the United States in the profession.

In 1920 Professor Westergaard had presented his interesting, though less significant, paper entitled *On the Resistance of Ductile Materials to Combined Stresses in Two or Three Directions Perpendicular to One Another*, which was published in *Journal of the Franklin Institute*, Vol. 189 (1920). This paper is the first discussion and evaluation of the pioneer work on failure theories by Rankine, Saint-Venant, Coulomb (1773), Guest (1900), A. J. Becker, W. Scoble, C. A. Smith, L. B. Turner, W. Mason, B. P. Haigh, and others; and gives references to their original contributions. Later discussions of these theories may be found in *Theories of Failure of Materials* by A. Nadai, in *Trans. A.S.M.E.*, Vol. 55 (1933), and in *Failure Theories of Materials* by J. Marin, in *Trans. A.S.C.E.*, Vol. 101 (1936).

Theory of Elastic Stability Brought to America.—Professor Westergaard's papers were followed in 1924 by one from the pen of Professor Stephen Timoshenko, who was then a recent arrival from Russia. This was entitled *Beams without Lateral Support* and was published in *Trans. American Society of Civil Engineers*, Vol. 87 (1924). It was, the author believes, Professor Timoshenko's American debut. It was an attempt to bring the European-developed principles of Elastic Stability before the American engineer. The fact that only one person, Mr. Joseph S. Newell, offered written discussion does not indicate that the paper was not important or well written but probably does indicate a lack of knowledge of this theory in America.

First International Congress of Applied Mechanics.—The year 1924 produced the First International Congress of Applied Mechanics, which was held at Delft, The Netherlands. A preliminary meeting organized by Professor Th. von Karman had been held at Innsbruck, Austria. The Congress at Delft was made possible because of the influence and labor of Professors Biezeno and Burgers of Delft. This

influential body has since held meetings at Zürich, Switzerland, in 1926; at Stockholm, Sweden, in 1930; at Cambridge, England, in 1934; and a fifth at Cambridge, Massachusetts, U.S.A., in 1938.

At this time a new interest in the mechanics of soils was roused by a paper read at the Delft Congress in 1924 by Charles Terzaghi, then of Constantinople, later of Harvard. This was the beginning of a new science, which was to be known as Soil Mechanics and is an application of Strength of Materials and Hydraulics to the study of soil behavior.

Increased Interest in Theory Shown Among American Engineers.—The interest in theory was beginning to take hold in America. In *Trans. American Society of Civil Engineers*, Vol. 89 (1926), there appears *Secondary Stresses in Bridges* by Cecil Vivian Abo. Mr. Abo devotes 150 pages to a critical comparison of the various existing methods of computing secondary stresses in bridges.

Trans. American Society of Civil Engineers, Vol. 90 (1927), contains the first of several papers on indeterminate structures, *Moments in Restrained and Continuous Beams by the Method of Conjugate Points*, by L. H. Nishkian and D. B. Steinman. The object is to present a rapid and direct graphic solution of the relation expressed by the Theorem of Three Moments. The method is applicable to continuous beams, rigid portals, and other indeterminate frames. The paper and the written discussion by seventeen interested persons is spread over 206 pages. Professor Hardy Cross of Yale University says that it is an interesting contribution to the American literature in this field. The paper really gives the American engineer a symposium in concise form of the various methods of analysis of indeterminate beams and frames.

Plastic Flow of Solids.—The first paper on the theory of the plastic flow of metals to appear in the American literature was *On the Mechanics of the Plastic State of Metals*, by Dr. A.

Nadai, which was published in *Trans. of American Society of Mechanical Engineers*, Vol. 52 (1930). This paper not only contains Dr. Nadai's original work in this field but also gives an introduction describing the general theory of plastic flow.

The first attempts to write equations describing the plastic flow of metals were made by Saint-Venant in *Comptes rendus* at Paris in 1870–1871. He developed the theory for two-dimensional plastic flow for a twisted bar. This theory was further extended in 1882 by C. Dugaet in *Limite d' élasticité et résistance à la rupture*, Vol. 1, page 157; and in 1913 R. von Mises, Göttingen, also formulated mathematical theory of the plastic state. More recently Saint-Venant's work was extended and improved by H. Hencky and L. Prandtl in papers in the *Proceedings of the International Congress of Applied Mechanics* at Delft in 1924 and by Th. von Karman at the 1926 meeting of the same society at Zürich.

In 1931 Dr. Nadai's book *Plasticity* was published as an Engineering Societies Monograph. Before coming to the United States, Dr. Nadai published in Berlin in 1925 a notable work on the newer investigations of plates, called *Die elastischen Platten*, and also published in 1927 *Der bildsame Zustand der Werkstoffe*. There are also numerous papers in the literature under his signature.

Theory of Redundant Structures.—In 1932 Professor Hardy Cross published his now well known *Analysis of Continuous Frames by Distributing Fixed-End Moments* in *Trans. American Society of Civil Engineers*, Vol. 96 (1932). Professor Cross says that his method is "a method of successive approximations, not an approximate method." This is possibly the greatest single contribution to the available theory for the analysis of redundant structures. It brought forth thirty-seven written discussions covering 127 pages, while the original paper, itself, was only 10 pages long. Pro-

fessor Cross comments on this by saying that he "took the time to make the paper short." The method was called approximate by some, to which Professor Cross replied: "The quest of the absolute is a beautiful thing; but he who seeks in engineering analysis a precision that cannot be ultimately translated into such units as pounds of steel and yards of concrete is misled."

Timoshenko's *Theory of Elasticity*, published in 1934 as an Engineering Societies Monograph, was an especially important contribution to the literature in this field in the English language, both because of its scope and value in engineering stress analysis and because it contains nearly 400 references, approximately half of which are to foreign authors.

The Theory of Thin Shells.—In 1934 W. Flügge, a member of the Göttingen school, published his important *Statik und Dynamik der Schalen*. This book gives in a clear and competent manner the mathematical theory of the equilibrium, vibration, and stability of thin curved shells. It is an excellent companion book to *Die Elastische Platten* published about ten years earlier by Dr. Nadai while he was at the same institution. Flügge's book gives a complete account of the extensive modern development of the theory of thin plates, pressure vessels, boilers, oil tanks, and airplane fuselages, and includes an excellent bibliography of the field.

Three-Dimensional Photo-Elastic Stress Analysis.—The year 1935 witnessed the entrance of the photo-elastic technique into the field of three-dimensional stress. James Clerk Maxwell, in a paper *On the Equilibrium of Elastic Solids* published in *Trans. of the Royal Society Edinburgh*, Vol. 20 (1853), was the first to recognize the property which makes three-dimensional stress analysis possible; but in *Mechanical Engineering*, Vol. 57 (1935), page 767, A. G. Solakian published the results of a torsional experiment made on a marblette rod, and this experiment seems to be the first work in

the development of the new three-dimensional technique. G. Oppel, in *Polarisationsoptische untersuchung raümlicher Spannungs und Dehnungszustände* in *Forschung auf dem Gebiete des Ingenieurwesens*, Vol. 7, No. 5 (1936), page 240, was the first to state that cutting the sample does not disturb its fringe pattern. No experimental proof was given, however. R. Hiltscher and A. Kuske of the same Munich school also made some contributions to the development of the new technique, but it was not until the publication of *Photo-Elastic Studies of Three-Dimensional Stress Problems* by M. Hetenyi, at the Fifth International Congress of Applied Mechanics at Cambridge, Mass., in 1938, that proof and explanations of this new technique were available. He also was the author of *Fundamentals of Three-Dimensional Photo-Elasticity*, which was published in *Journal of Applied Mechanics, Trans. American Society of Mechanical Engineers*, Vol. 60 (1938).

In *Trans. American Society of Civil Engineers*, Vol. 101 (1936), page 423, there is a paper by D. H. Young entitled *Rational Design of Steel Columns*, which again set off important discussion of this—since Euler—controversial subject. The paper and its discussion should be read by all engineers interested in column theory. The same volume contains on page 857 an interesting paper entitled *Structural Beams in Torsion*, by Inge Lyse and Bruce G. Johnston; and another on page 1162 entitled *Failure Theories of Materials Subjected to Combined Stress*, by Joseph Marin, which gives a good summary of the various theories of failure and considerable reference material.

Theory of Approximate Computation.—The year 1936 also witnessed the publication of R. J. Southwell's *An Introduction to the Theory of Elasticity for Engineers and Physicists*. This is definitely a distinguished book in the English tradition by a distinguished English scholar. It is

not just a reference book nor strictly a textbook, but is rather a book for the mature engineer or physicist to read. As the title indicates, the book does not completely cover the field but is selective. Naturally it follows the English tradition a bit more than the American engineer will like, but it is a book which should be read by all Americans who wish to be well informed on the Theory of Elasticity. Professor Southwell is also the author of *Stress Calculation in Frameworks by a Method of Systematic Relaxation of Constraints*, published in *Proceedings of the Royal Society of London*, Series A, Vol. 151 (1935), p. 56; and in 1940 he published a book *Relaxation Methods in Engineering Science*, with a subtitle *A Treatise on Approximate Computation*, which discusses his efforts over nearly five years with this technique. In *Proceedings of the Royal Society of London*, Series A, Vol. 169 (1938–1939), p. 476, there is another paper on this technique, entitled *The General Theory of Relaxation Methods Applied to Linear Systems*, by G. Temple.

More About Redundant Structures.—Ideas important to engineering philosophy, as well as to structural design, were expressed in a paper entitled *Theory of Limit Design*, which was published in *Trans. American Society of Civil Engineers*, Vol. 105 (1940), p. 639, by Professor J. A. Vandenbroek of the University of Michigan. He attempts to show that the limit of the strength of a redundant structure, or its capacity load, is arrived at only after as many members of the structure corresponding in number to the structure's redundants have all reached their elastic or buckling-limit strength. He also shows that this limit of strength can be easily calculated and in many cases is considerably different from the strength calculated by more conventional methods. That this paper is on a highly controversial topic, and also is one which may well be studied by qualified engineers before they express final judgment regarding the merit of the theory

advanced, is evident from the interest shown and the eighteen written discussions. Several Europeans have also written on the subject. The references are given on page 661 of Professor Vanderbroek's article.

Brittle-Lacquer Stress Analysis.—The *Journal of the Aeronautical Sciences*, Vol. 7 (1940), contains an article *Brittle Lacquers as an Aid to Stress Analysis*, by A. V. DeForest and Greer Ellis of the Massachusetts Institute of Technology. This paper describes a new technique of stress analysis which these gentlemen have since developed to a high degree of success. The first papers on this subject were: *Das Dehnungelinierverfahren, Zeitschrift des Vereines deutscher Ingenieure* for 1932, by Dietrich and Lehr; *Procédé d'Étude de la Distribution des Efforts Élastiques dans les Pièces Métalliques*, by Portevin and Cymboliste, in *Revue de Métallurgie*, Vol. 4 (1934); and *Strain Indicating Lacquers*, Aero Engineering Department, Massachusetts Institute of Technology, 1937.

Energy Methods.—In *Trans. American Society of Civil Engineers*, Vol. 107 (1942), p. 765, we find another important paper by Dean H. M. Westergaard, this one being entitled *Method of Complimentary Energy*. The basic law underlying this method was first stated by F. Engesser in *Zeitschrift des Architekten und Ingenieur-Vereinszer*, Hannover, Vol. 35 (1889), but this paper did not become well known. Engesser extended Castigliano's law of least work to apply beyond the limits of Hooke's law. Castigliano's work is found in *Atti della Reale Accademia delle Scienze di Torino*, Vol. 10 (1875), p. 380, and is further developed in *Théorie de l'équilibre des systèmes élastiques et ses applications*, published at Torino in 1879, page 39. The method was also improved and made better known through the work of H. Müller-Breslau entitled *Die Neueren Methoden der Festigkeitslehre und der Statik der Baukonstruktionen* and published at Leipzig in 1886. Another useful and critical discussion of the original

work in this field was written by M. Grüming in 1912 in his paper *Theorie der Baukonstruktionen,* which was published at Leipzig in *Encyklopädie der Mathematischen Wissenschaften,* Vol. 4, sub-volume 4 (1907–1914), page 419. Professor Westergaard's work adds much to these earlier discussions.

CHAPTER 14

Mechanics and Engineering Education in the United States

The Birth of Engineering Education.—The picture which we have been trying to create would be decidedly lacking in detail if the story of the development of formal engineering education and the parallel progress of Mechanics in America were neglected. To tell this story it is necessary again to return to Europe and examine the educational situation there.

The European pioneers of our two sciences were largely theorists and individualists who apparently were not too greatly interested in the practical application of their knowledge and talents, probably because man's mode of life then existent presented few problems that required the efforts of a Galileo, a DaVinci, a Newton, or even one of our lesser satellites. Such men were permitted to devote their talents to matters which interested them, undisturbed by the problems of the ordinary citizens. The demand for bigger and better gadgets then was not what it is today.

The creators of the structures of antiquity and the Renaissance were both architects and engineers, but when and where science and construction began to unite to form what is now known as engineering is not exactly known. It probably was in France where the royal *Corps des Ponts et Chaussées* is known to have existed during the reign of Charles V (1364–1380) who mentioned *nos ingénieurs des ponts et chaussées* in a royal communication, according to the Report of the Investigation of Engineering Education, S.P.E.E., 1923–1929, Vol. 1, p. 758.

In 1747 Louis XV of France appointed Perronnet, who has been called the father of modern civil engineering and

engineering education, chief engineer of bridges and highways. He was made responsible for "the direction and supervision of surveyors and designers of plans and maps of the roads and highways of the realm and of all those appointed and nominated to said work; and to instruct the said designers in the sciences and practices needful to fulfilling with competency the different occupations relating to the said bridges and highways."

Here was the authority required for the establishment of a school for engineers, and Perronnet immediately (1747) set up for his staff an organization which required systematic study and provided rewards for successful accomplishment. The school thus established in 1747 on a basis of "mutual instruction," assisted by such external professors as Perronnet was able to provide, continued to function successfully; and in 1775 it was legally named *École des Ponts et Chaussées*, the first school of formal engineering education. This was followed in 1765 by a royal *Bergakademie* at Freiberg, Saxony, and in 1778 by a school of mineralogy, assaying, and metallurgy at Paris.

During the Revolution the French technical schools which supplied civil and military engineers to the state became badly disorganized. It was found that the entering students were not well prepared. To correct this defect *École des Travaux Publics* was established in 1794. This institution was to train engineers for both public and private service. Entrance was by competitive examination. It was replaced a year later by *École Polytechnique* which was fortunate in attracting to its staff such men as Lagrange and LaPlace in mathematics, Prony in mechanics, and many other able men of science. Since its beginning the *École Polytechnique* has maintained its standards so that it has become the most desirable, yet the most difficult to enter, of all the French schools. These standards were maintained by strict limitation of the student body, careful selection of students, and

rigorous and closely supervised curricula taught by an eminent faculty.

Early English Engineering Education.—What was the status of engineering education in England during these early days of the application of science to industry? A glance at the names now credited with being the inventors of the devices which brought about the industrial revolution in the industry of that country gives the answer. Newcomen, the steam-engine man, was an iron monger; Crompton and Hargreaves were weavers; Smeaton and Watt were instrument makers; and Stephenson was a fireman who learned to read as a grown man. What sort of science could possibly spring from such soil? True, there were the universities with their men of learning and drawing their scholars from the leisured classes, but they had no connection with industry and desired none.

From the time of Elizabeth until 1814 when the Elizabethian laws were repealed, education for the trades or professions was by the indenture system which required an apprenticeship of seven years for entrance into any trade or profession. With the repeal of this act, new horizons of advancement were opened up. The Institution of Civil Engineers, founded in 1818, was created as a means of mutual education to supplement the training by the pupilage system which had become the accepted form of entrance into the engineering profession. The pupilage system was on a slightly higher level than the apprenticeship system because the pupils paid a fee, were supposed to have had an ordinary grammar-school education, and usually were about seventeen years of age. The training which the pupil obtained was, however, too dependent on the ability, training, and ethical standards of the engineer who accepted the pupil. The Institution of Civil Engineers recognized the weakness of this sort of training, but did not do anything to guarantee the

qualifications of its membership until almost a century later (1897), when it introduced a system of entrance examinations.

As the industrial plants grew larger and more specialized, the pupilage system began to decline and a system of apprenticeship supposedly coordinated with schooling came into use. The spirit of the early English system may be obtained from a presidential address of Sir Benjamin Baker before the Institution of Civil Engineers as late as 1895 in which he said: "If anything more remains to be said on the subject, it must therefore be of the nature of a warning, that technical education is of little value unless accompanied by the practical experience, sound judgment and bold initiative, which, rather than book knowledge, characterized the famous members of the Institution in the past."

The Royal Technical College of Glasgow, the oldest school of applied science in the English-speaking world, owes its existence to a gift made in 1796 by John Anderson, Professor of Natural Philosophy at the University of Glasgow. Anderson's idea was to make scientific studies and methods available to workmen. Although the poor preparatory education of the workers became a serious handicap, the idea did furnish the seed for other technical colleges.

This was the state of English technical education at the beginning of the nineteenth century. At the top were Oxford and Cambridge, which were chiefly occupied in imparting literary culture to the leisure classes but had no great interest in scientific investigation or its application to industry. At the bottom was the apprenticeship system for trade education, and in between was a great gulf which a relatively small group were trying to fill by training their successors through an inadequate system of pupilage.

It was not until after the middle of the nineteenth century that any real progress in higher engineering education at the university level was made. Oxford and Cambridge offered

little physical science before 1850. The present English system owes much to the Scottish universities and to such men as Rankine and Kelvin.

Beginning of Technical Education in Germany.—In Germany, Prof. Eberhard lectured on applied mathematics at Halle in 1773; and at Heidelberg and Göttingen attempts were made to introduce technical subjects into the curriculum, but these efforts were unsuccessful. The exponents of academic culture would not permit their disinterested learning to be contaminated with anything so practical.

Previous to 1800 apparently the only institution offering technical training was a royal *Bergakademie* founded at Freiberg in 1765 by Prince Xaver for the teaching of mining and metallurgy. This was followed by the royal *Bauakademie* at Berlin, which was really a rather elementary school for surveyors and constructors and admitted students fourteen years of age with only a common-school education. In 1806 a polytechnic school was opened at Prague, and in 1815 the Polytechnic Institute of Vienna was established. Both these schools required only arithmetic for admission and accepted students fourteen years of age; in fact, Prague and Vienna started students at the age of thirteen, if they entered a preparatory course. Such institutions could hardly be classed as anything more than trade schools.

This constituted the German technical education program until the polytechnic school was established at Karlsruhe in 1825. In 1833 Karlsruhe was reorganized into a professional school with courses for architects, civil engineers, and foresters. It thus became Germany's first real technical school.

Early Practical Science Education in the United States. It was with a background of precedents such as we have described that the system of engineering education in America was born. The colleges which had been established during the colonial period were exclusively classical in their

traditions and followed the Oxford and Cambridge pattern. Only a few made any attempt at teaching the physical sciences. This was the situation which faced the men who realized that if America was to exploit its natural resources the youth of the country had to have the help which could only be obtained through technical training.

The engineers of the day were a small group of land surveyors and practical self-taught constructors of public works and machinery. There was no established system of pupilage, as in England. The need had to be supplied by other means. The engineering college was the answer to the problem, since the established colleges would have no part of such training.

The first attempt to teach practical science was at the Gardiner Lyceum (1822) at Gardiner, Maine. This school was financed by R. H. Gardiner and other people of the town, with some assistance from the state legislature. However, the legislature soon withdrew its support and the school closed.

A year later (1823) Stephen Van Rensselaer, a wealthy land owner of Albany, established at Troy a school "for the purpose of instructing persons who may choose to apply themselves in the application of science to the common purposes of life." This school was under the direction of Amos Eaton. He was a shrewd Yankee "jack of all trades," a sort of combination civil engineer, geologist, chemist, botanist, and lawyer, who developed into a successful educator. In twelve years Rensselaer became the first professional school of civil engineering in the English-speaking world.

Applied Science at West Point.—The United States Military Academy graduated its first student in 1802, two decades before Gardiner and Van Rensselaer started their schools for the public. In its early years it apparently was a corps of engineer cadets. It was not until 1817, when Col. Sylvanus Thayer became Superintendent after studying in

Europe, that the Military Academy began to reorganize its courses and give its students the mathematical training which soon set the standards for professional engineering education in the United States.

As a result of Col. Thayer's European observations, the West Point curricula followed the French pattern of rigorous theoretical and mathematical training. It is through the establishment of these curricula that West Point became the first American school of applied science. Rensselaer granted its first four degrees of Civil Engineer in 1835.

During the early years at West Point, according to the Annual Report of the Superintendent for 1896, Enfield's *Institutes of Natural Philosophy* was used as a Mechanics text. It would appear that even in these early days there were good students and students who were not so able, because in 1818 Olinthus Gregory's *Treatise on Mechanics* was adopted and a year later Parkinson's *Mechanics* was taken on as a text for the second group. Evidently the ebb and flow of textbooks was about the same as at the present time for in 1824 Bridge's *Treatise on Mechanics* replaced Parkinson and six years later (1830) *Traité Elémentaire de Mécanique* by Francoer replaced Gregory and Boucharlat's *Traité de Mécanique* was adopted for the less able students. These books were in the original French. We wonder how our present-day students would receive a French text. However, the West Pointers were not forced to carry this additional burden long, for in 1833 an English translation of Boucharlat by Edward H. Courtenay, Professor of Natural and Experimental Philosophy at the Academy, appeared. This book seems to have been the first Mechanics text for technical students to be printed in the United States. The book continued in use at the Academy until 1850 when, according to the same report, the Academy adopted *Synthetical Mechanics* by W. H. C. Bartlett, L.L.D., Professor of Natural and Experimental Philosophy at the Academy from 1834 to 1871.

This book was practically a translation of an earlier work by J. V. Poncelot. Apparently the geometrical approach of the book did not please the staff at the Academy, for three years later (1853) it was discarded in favor of *Elements of Analytical Mechanics*, also by Professor Bartlett. This book was eminently successful and continued in use at the Academy for over three decades or until 1887 when Peter S. Michie's *Elements of Analytical Mechanics* replaced it.

The First American Mechanics Textbook.—Bartlett's *Analytical Mechanics* is probably the first American textbook devoted strictly to Mechanics. There are other earlier books, such as *An Introduction to Natural Philosophy* by Denison Olmstead, L.L.D., Professor of Natural Philosophy and Astronomy, which the author wrote in 1831 for his classes at Yale and which passed through many editions and was used in many colleges throughout the country for more than sixty years. However, this book devoted only 246 pages to Mechanics and 13 pages to Strength of Materials, and the rest of its 592 pages were given over to what now generally passes as physics. It was patterned after the early English books but did introduce a feature now common to most American Mechanics textbooks—problems for student solution.

Bartlett's book was strictly a Mechanics textbook covering most of the topics found in the modern textbook and much that is not now taught in the engineering or technical type of Mechanics course. The eighth edition, published in 1873, had the following contents:

Part 1—Mechanics of solids, pages 31–261
Part 2—Mechanics of fluids, pages 263–338
Part 3—Mechanics of molecules, pages 345–400
Part 4—Applications to simple machines, pages 405–480

In the introduction to the second edition (1858), the author says: "It now appears with an additional part under

the head of Mechanics of Molecules and this completes—in so far as he may have succeeded in its execution—the design of the author to give to the classes committed to his instruction in the Military Academy what has appeared to him a proper elementary basis for a systematic study of the laws of Matter."

Bartlett used calculus extensively and solved many special problems, but he dealt mostly with symbols and apparently did not favor Olmstead's idea of problems for student solution, for none of these are included. The book is exceedingly abstract, with the mathematical theorist's approach rather than that of the practical engineer. The language of the text is difficult and involved. Such is the text from which the West Pointers studied their Mechanics for thirty-three years.

Influence of Morrill Land-Grant Act on Engineering Education.—The ever-increasing use of power for transportation and in manufacturing produced a greatly increased demand for technically trained engineers. This and the passage of the Morrill Land-Grant Act in 1862 resulted in a rapid increase in the number of colleges offering engineering instruction.

The Wickenden Report of the Investigation of Engineering Education (1923–1929), Vol. I, page 542, contains the following statement: "In 1866 there were but six engineering colleges of established reputation and but 300 men had been graduated in the previous thirty-one years. By 1870 . . . the total number of engineering colleges had mounted to twenty-one and the number of engineers graduated to 866." This indicates the early numerical weakness of the engineering profession. Also, some idea of the character of engineering education in general and the instruction in Mechanics in particular prior to 1870 can be obtained from the following quotation from the same report, pages 543 and 544:

"The great masters of mathematical exposition set a fashion of teaching which was criticized as too severe and impractical. School facilities being inadequate, and class laboratories almost unknown, the results of experimental research were dressed in mathematics and taught largely through the weight of authority from the book; and while the instructor performed elementary experiments on the other side of the table, the class simply observed and perhaps tried to take notes."

Such practices led Prof. Mansfield Merriman to make the following comments in his presidential address before the Society for the Promotion of Engineering Education in 1896:

"Thirty years ago public opinion looked with distrust on technical education. . . . Engineering courses of a quarter of a century ago were scientific rather than technical. It was recognized that the principles and facts of science were likely to be useful in the every-day work of life and particularly in the design and construction of machines and structures. Hence mathematics was taught more thoroughly and with greater regard to practical applications. . . . Although engineering practice was rarely discussed in those early schools . . . yet the scientific spirit that prevailed was most praiseworthy and its influence has been far reaching.

"This scientific education notably differed from the old classical education in two important respects: first, the principles of science were regarded as principles of truth . . . and second, the laws of the forces of nature were recognized as important to be understood in order to advance the prosperity and happiness of man. . . . Gradually the latter tendency became far stronger than the former and thus the scientific school developed into the engineering college."

Such rapid growth of the number of engineering colleges naturally encouraged the writing of textbooks, especially Mechanics textbooks. There now was a demand for a text

with a specific purpose, not just a book written to furnish the student with material for mental exercise.

Mechanics Texts.—In 1870 Prof. W. G. Peck published his *Elementary Treatise on Mechanics,* a book which contained no calculus and which according to the author was an attempt to produce a book that would serve those who did not wish to pursue the more difficult books. However, Prof. Peck's book was not too well written in its technical detail.

This book was followed in 1876 by *The Elements of Analytical Mechanics* by DeVolson Wood, A.M., C.E., who was an educator with fifteen years of experience as a teacher of engineering at the University of Michigan and who in 1872 became Professor of Mathematics and Mechanics at the Stevens Institute of Technology. Wood's book was printed in several editions and, while extremely mathematical and theoretical in places, it did have some engineering approach and contained many problems for student solution, including some on trusses and frames.

In his introduction Wood says: "I have avoided the use of the terms Force of Inertia, Impulsive Force and Instantaneous Force except to refer to them and define the senses in which they are generally used; for they are not only useless, but harmful. . . . I do not believe that inertia is a force, and in this I am supported by many eminent writers; but there are those, also eminent, who state distinctly that it is a force."

In 1884 E. A. Bowser, L.L.D., Professor of Mathematics and Engineering at Rutgers, published *An Elementary Treatise on Analytical Mechanics.* This successful book was theoretical and highly mathematical in nature, but the author did introduce over 600 unsolved problems; and the inclusion of these problems may have been the reason for the book's popular appeal over almost forty years. The author

apparently did not consider framed structures part of Mechanics for he introduced no truss problems.

Bowser's book was followed in 1885 by *Applied Mechanics* by Gaetano Lanza, S.B., C.E., and M.E., Professor of Theoretical and Applied Mechanics at the Massachusetts Institute of Technology. In his introduction the author says: "The work is essentially a treatise on strength and stability; but it was thought best to call it Applied Mechanics notwithstanding the fact that a number of subjects really included in treatises on Applied Mechanics are omitted."

In this voluminous book of 979 pages, only 221 pages are devoted to material usually classified as Mechanics, while the remainder is on strength and stability. Lanza treats trusses by both the analytical and graphical methods. He uses the method of sections but does not call the isolated body a "free-body."

In 1887 Prof. Peck brought out his second text, which was called *Elementary Treatise on Analytical Mechanics*. This book was a bit more advanced than the earlier one in that it employed calculus and contained a number of problems for student solution.

The Beginning of the Practical Treatment of Mechanics. With the publication in 1887 of *Mechanics of Engineering* by Irving P. Church, C.E., Professor of Applied Mechanics and Hydraulics at Cornell University, the American Mechanics textbook began to break away from the theoretical treatment so favored by the mathematicians and philosophers and to present the subject in a more practical manner. Prof. Church apparently was the first to call his isolated bodies "free-body" diagrams, and he also designates simple tension and compression members as "two-force pieces."

Church, like Lanza, attempted to cover the entire field of Mechanics in one book. Of the over 850 pages, only 186

pages are devoted to Statics and Kinetics and the remainder covers Strength of Materials and Hydraulics.

Since the turn of the century many additional books have been published, and there was a continuation of the trend which started when DeVolson Wood began to shift away from virtual velocities in favor of the methods of force, mass, and acceleration, impulse and momentum, and work and energy. Much material of a theoretical and mathematical nature has been dropped as the teaching of Mechanics in the engineering colleges was taken over by men who had the professional engineer's viewpoint rather than that of the mathematician or philosopher, as was the case in the early days. Problems of a practical nature were added in large numbers for student solution. Such aids to analysis as the "free-body diagram" were freely employed, along with simplification of the mathematical processes. Mechanics was converted from a science of interest only to able mathematicians and philosophers into an implement of service to the practical engineer.

Strength of Materials Becomes Part of the Engineering Curriculum.—While we have observed that Galileo had something to say about Strength of Materials in 1638 and many others had been interested in various phases of its mathematical theory, Strength of Materials received little attention in the curricula of the newly established engineering colleges in America until after 1870. There were two major reasons for this: First, instruments for making accurate measurements and machines for performing physical tasks were not available until after the beginning of the nineteenth century. Second—and probably this is the more important reason—there was no great demand for such knowledge and instruction until after steel was made available in quantity and at low cost by the Bessemer process, adequate power, and efficient rolling equipment.

The early Mechanics texts usually devoted about a chapter of twenty to thirty pages to the theory of simple wood beams. This limited treatment was based on pure rationalization unchecked by accurate experimentation. It was all that was considered necessary in a day when building and construction was largely the business of the practical man.

We have observed the European progress from Galileo's crude beginning on down through the years. Now let us see how America fits into the picture. Again we must turn to the Military Academy. Here we find that apparently the first attempts at formal instruction in the theory of Strength of Materials were made in 1818, when Gregory's *Treatise on Mechanics* was used as the text. This book devoted a single chapter of twenty-nine pages to the discussion of beams. In this limited manner was the subject introduced to the American student. The quality of the book has already been commented on.

By 1823 the Academy staff realized that its graduates were going to have to cope with construction problems and introduced a textbook by M. I. Sganzin on engineering construction which two years later was translated into English and called *An Elementary Course in Civil Engineering*. This book was largely of a descriptive nature but did devote some space to Strength of Materials and the strength of beams.

In 1830, with the adoption of Boucharlat's *Treatise of Mechanics*, the subject of Strength of Materials had attained the dignity of an additional three pages or a total of thirty-two pages but was still limited to a discussion of rectangular beams, although Boucharlat's theory was considerably more accurate than Gregory's.

The year 1830 was also important because it brought Dennis H. Mahan, a 1824 graduate, back to the Academy as a teacher of engineering after study in France. The influence of Mahan's French training on the engineering instruction at

the Academy and at other institutions throughout the United States where his engineering textbooks were used was of great importance. In 1837 he published *An Elementary Course in Civil Engineering*, in which he devoted a chapter to materials of construction and presented formulas for beam and column design.

This was the status of Mechanics of Materials in the United States until—after the Morrill Land-Grant Act—important expansion of the railroads, the successful application of steam power to manufacturing, and the cheap and abundant production of steel by the Bessemer process made it necessary that those who designed new structures do so not according to instinct or "rule of thumb" but according to the fundamental principles of Mechanics and experimentally established knowledge of the strength of the materials used. The transition from the old era to the new began about 1870.

The First American Strength of Materials Textbook.—In 1871 DeVolson Wood, then Professor of Civil Engineering at the University of Michigan, published what is now considered to be the first American textbook on Strength of Materials, this book being based on his lectures as given to his students. He comments in the following manner on the strength of rectangular beams:

"The 'Common Theory', as I have called it, is sufficiently correct for ordinary practical purposes especially if the modulus of rupture, as determined by direct experiment upon rectangular beams of the same material, be used. Barlow's theory of flexure appears to be more clearly correct in theory when applied to rectangular beams and beams of the I section, or other forms which are symmetrical in reference to the neutral axis. . . . Whatever theory may yet do for us it is quite evident that no theory will ever be devised, of practical value, which will be applicable to the variety of forms of beams which are or may be used in mechanics arts."

So says Prof. Wood in 1871. Then he continues with: "If a theory is ever devised which will take into account all the conditions of strains in a beam I think it will be too complicated to be of practical value to the mechanic."

Wood's work is a remarkably well written book of 245 pages, which every serious student of Mechanics of Materials would do well to examine. There are many historical references contained in the text. Prof. Wood produced an excellent book for such an early date.

American Engineering Attains Professional Status.—The next important contribution to the literature was *The Elasticity and Resistance of the Materials of Engineering* written in 1883 by W. H. Burr, C.E., Professor of Rational and Technical Mechanics at the Rensselaer Polytechnic Institute. The first 207 pages of this 753-page book are devoted to what the author called the Rational part of the work—the mathematical treatment. Of this he says: "It will undoubtedly impress a great number and perhaps all engineers in active practice, that it is unnecessary to the proper treatment of such a subject. Indeed a very extended experience in iron and steel constructions places the author himself in position to fully appreciate the weight of such a criticism at the first glance. But it may be contended and he thinks must be admitted, that the present advanced state of engineering as a profession implies the existence of something that may be called the 'natural philosophy' of engineering. In other words the engineer of the present time must meet the increased and increasing demands upon him in some one or more specialty, not only by the aid of common sense and a well trained judgment, but also by a systematic knowledge of much of natural philosophy as is involved in practical engineering operations." Here is a pertinent indication that engineering was, at least by some, realized to be no longer "cut and try." Burr also says: "An

engineer's preparation for active practice must consist both of the philosophical training in what is largely ideal and which he acquires in the technical school and the purely practical training of the first few years of his professional life."

These statements by a recognized educator indicate that, in the United States at least, engineering and its application of science to the practical things was on the way to becoming a profession no longer to be dominated by the men who had just grown up in the construction industry.

The remaining 546 pages of Burr's book, called Technical, consisted of the compilation and analysis of a large amount of experimental data from many sources.

Burr's book was followed in 1885 by that widely used small (150 pages) but important book *Mechanics of Materials* by Professor Mansfield Merriman of Lehigh University. Professor Merriman uses the technique, now quite generally adopted, of breaking the material up into a series of articles each of which is devoted to a particular part or phase of a subject, and then generally introducing a problem or two for student solution. He uses moment diagrams and shear diagrams; and he may be the originator of the shear diagram, but this fact has not been definitely established. In the earlier editions he apparently was not aware of the relationship existing between the shear diagram and the bending-moment diagram, because the calculus method of determining the location of maximum bending moment for distributed loads is used. In later editions the relationship between the shear and bending-moment diagrams was described.

Also in 1885 Lanza's *Applied Mechanics*, the all-purpose book already mentioned, appeared. In the portion devoted to the resistance of materials the subjects of direct stress, torsion, and bending of beams and columns are discussed, and much of the more advanced theory is also included; but, since relatively few problems for student solution were intro-

duced, the book was rather unsatisfactory, except as a reference work.

In 1887 the other pioneer all-purpose book, Church's *Mechanics of Engineering*, was published. This book was much more successful as a text, probably because it contained many completely solved numerical examples as well as a sizable number of problems for student solution.

After these publications by Burr, Lanza, Church, and Merriman, Mechanics of Materials became an established well defined course in the curricula of the engineering colleges of the United States. It had been taken out of the hands of the philosophers who had accorded it but a brief and superficial treatment; had passed through the stage where such men as Burr found it necessary to justify the value of sound theory as a supplement to the knowledge of practical builders; and had now become an acknowledged tool which every technically trained engineer must possess.

Changes in Administrative Relations.—It is interesting also to note the administrative changes which were, at least in part, responsible for the present stature of our two sciences. During the last seventy-five years the administrative responsibility for these courses has passed from the department of natural philosophy first to the mathematics department and then to the civil-engineering department or in a limited number of cases to the mechanical-engineering department; and, finally, during the last twenty-five years at most of the larger engineering colleges these sciences have attained the dignity and authority of an independent department or school, which in some cases is recognized as sufficiently mature to grant both the bachelor's degree and a graduate degree.

Mechanics, in its various phases, is no longer a subject used by philosophers and mathematicians for demonstrating their analytical ability, but it is now recognized and estab-

lished as a powerful implement of the engineering profession. America's most important contribution to this growth has been in simplication and organization, so that Mechanics has become a tool usable by many instead of a phase of natural philosophy comprehensible only to a few with exceptional analytical and mathematical ability.

CHAPTER 15

The Nineteen-Forties

As we moderns pick up a dial telephone and talk to a friend or business associate in a European capital or watch a ball game which is being played on a field miles away, and we then turn to a magazine which describes some projected train or supersonic airplane that is to be driven across the continent in a few hours by the atomic energy contained in a small quantity of matter, it is difficult to maintain our mental equilibrium. It is not at all surprising that we become a bit contemptuous about the accomplishments of our predecessors of a few decades back. We forget that it was the spade work of these brilliant and courageous men which prepared the ground for the foundation stones supporting all our accomplishments. It has been said that "history is a plaything of old men, and that young men have too much to do to keep up with the accomplishments of the past." That may be partially true. However, the more young men the world possesses who can become well enough educated to be called old men in their youth, the fewer wars and other great man-made catastrophes we shall have.

The real purpose of this book is to put the essentials of the story of the growth and maturity of that portion of our scientific knowledge known as Mechanics—and its now virile off-spring, Mechanics of Materials—into such readable form as to make it available to our young men and possibly stimulate their curiosity.

William Barclay Parsons, in his *Engineers and Engineering in the Renaissance*, says: "In the field of theory of design there was but little accurate knowledge. Engineers relied entirely on experience and judgment when they came to fix the dimensions of structural parts. There was no means of

testing materials to determine their resistance to strain. There were excellent mathematicians, the principles of geometry had been known for centuries. Experiments could easily have been made, but until the last years of the period no one appears to have thought that stresses in structures were subject to physical laws." Leonardo da Vinci apparently had some thoughts in this direction, but the casual notes he made were not published until others had made such progress that his work was valueless, except historically.

As the writer now steps aside and attempts to obtain a comprehensive picture of the period of approximately 2500 years in which our sciences matured, he is immediately aware that the most important factor—to him at least—is the personality of the workers. It is true that many men made what we consider important contributions to the gradual growth of the theory, but those advances made by men who possessed great ability as teachers or leaders seem to be the accomplishments we now consider the most vital. The influence of the gifted teacher who can attract sizable numbers of exceptionally able students seems to be much greater than the influence of the genius without that trait of personality.

We Americans can claim no part in the discovery or perfection of the basic Theory of Mechanics or of Mechanics of Materials, possibly because the Europeans had done a pretty good job even before the western hemisphere had ceased to be a wilderness inhabited by Indians and wild animals. Previous to the beginning of the present century there was no indication of the existence of any intellectual aristocracy in science in the western hemisphere. Apparently, such an aristocracy is at least one of the essential requirements if fundamental theory is to be produced in any country.

Some will say that, since we had our scientific and engineering societies and numerous colleges and technical publications, there must have been some great men. We know that there were successful leaders and men with great talent

and ability for making decisions and taking aggressive action, but there seemed to be few men, if any, who were able to do sufficient original thinking along mathematical or scientific lines to leave something of importance behind. A casual examination of the proceedings of our technical and engineering societies will quickly justify this statement. Very little in the way of important writing was produced in this country prior to World War I. A comparison of the papers from 1900 to 1920 with those from 1920 to 1947 immediately indicates that the writers of the later period had been subjected to a vitalizing experience which those of the earlier generation had not had.

Just what was it then that brought about this change? To this writer it appears to have been the infiltration of foreign-born and foreign-educated scientists and engineers. Many of these men came to the United States for political and other reasons immediately after World War I. Their thorough theoretical training gave them a decided advantage over their American fellow-workers who were not long in realizing that, while they were superior to the Europeans in production work, they were decidedly lacking in mathematical and scientific training. Fortunately, there were several of these Europeans who had been teachers in their native countries and who were desirous of continuing in the profession.One of these men, Dr. Stephen Timoshenko, has had a profound and stimulating influence on the greatly increased productivity of American research workers and teachers during the past two decades through his numerous books and papers. Some of his books are: *Vibration Problems in Engineering* (1925), *Strength of Materials* (1930), *Theory of Elasticity* (1934), *Theory of Elastic Stability* (1936), *Engineering Mechanics* (1937), and *Theory of Plates and Shells* (1940).

His greatest influence, however, has been through his ability to attract and stimulate both mature men and able young American students. Dr. Timoshenko came to the

United States from Russia. After a short period in industry, he became Professor of Engineering Mechanics at the University of Michigan. He is another example of the great influence wielded by a great teacher. This influence is so far-reaching that it cannot be measured. Timoshenko's leadership, because of his international scholarship and analytical technique, has not been confined to the laboratory or classroom but has been strongly felt in the activities of such bodies as the International Congress of Applied Mechanics; the Applied Mechanics Division of the American Society of Mechanical Engineers and its *Journal of Applied Mechanics*, to which he has contributed greatly by assisting in guiding its editorial policy; the American Society of Civil Engineers; and the Eastern Photo-Elastic Conference, which has recently expanded into the broader-based Society for Experimental Stress Analysis. All these societies have made important contributions in bringing the United States to its present position of leadership in the application of the fundamental principles of this highly theoretical science of Mechanics of Materials.

We must concede that Europe, especially France, was the leader in the pioneer days and that England, Germany, and Italy also contributed to the formulation of the basic principles. But to the United States, since the turn of the present century, must be given the crown for leadership in the practical application of these basic principles.

The stimulus has been furnished by the insistent demands of our higher scale of living, which was made possible by the initiative and daring of our much-maligned business men. These men have been willing to risk their capital and established businesses to satisfy the demands of our new and progressive country. Under the capitalist system they were permitted to develop large financial resources, making possible the building up and continuation through good periods and bad, under university-trained men, of large-scale

well-organized research programs. Here is probably the real key to our present leadership. Let us hope that the people of this great country of ours will, in the difficult times ahead, have the wisdom to continue in power a democratic form of government which will give capital and science the incentive and privilege to continue on its successful path of free and unrestricted enterprise.

The future of America both in peace and in war lies to a great extent in the hands of American scientists and their source—the universities.

APPENDIX A

Chronologically Important Periods and Dates of Mechanics

3500 B.C. Early development of mechanical trades in Near East.
2500 B.C. Beginning of arithmetic.
600 B.C. Beginning of elementary surveying and geometry.
384–322 B.C. Aristotle develops his philosophy.
300 B.C. Geometry becomes a formal science with Euclid.
287–212 B.C. Archimedes develops his principle of the lever, center of gravity, and the principle of buoyancy; and does the first work on statics.
700 A.D. The Moors enter Spain.
1200 A.D. The Moors introduce Hindu arithmetic and Arabic algebra into Spain.
1214–1294 Roger Bacon, the prophet of scientific development.
1452–1519 Leonardo da Vinci: Statical moment, and also the germs of the concept of work and the impossibility of perpetual motion.
1473–1543 Copernicus: The new planetary system.
1548–1620 Stevinus: The principle of the parallelogram of forces, the inclined plane, and buoyancy.
1586 Publication of *Wisconstige Gedachtenissen*.
1564–1642 Galileo: The beginning of dynamics, the theory of falling bodies, the pendulum, the concept of force, and impact.
1638 Publication of *Discorsi e Dimostrazioni Matematiche*.
1639 Marcus Marci: Published *Proportione Motus*. First work on impact.
1596–1650 Descartes: Analytical geometry.
1629–1695 Huygens: The compound pendulum, center of oscillation, "vis viva," ΣMv^2, centrifugal force, and impact.
1673 Publication of *Horologium Oscillatorium*.
1642–1727 Newton: *Principia*, laws of motion, law of gravitation, centrifugal force, laws of impact, and fluxions.
1687 Publication of *Philosophie Naturalis Principia Mathematica*.
1654–1722 Varignon: Statics based on principle of moments. Publication of *Project d'une Nouvelle Mécanique* (1687).

1646–1716 Leibniz: A system of calculus.

1654–1807 The Bernoullis: Further development of calculus, equation of the catenary, and deflection of beams.

1707–1783 Leonhard Euler: Improved calculus and analytical mechanics.

1736 Euler published *Mechanica sive Motus Scientia Analytice Exposita*.

1717–1783 Jean LeRond D'Alembert: D'Alembert Principle (1743). Publication of *Traité de Dynamique*.

1736–1806 Charles Augustin Coulomb: Laws of plane friction.

1736–1813 Joseph Louis Lagrange: Further development of analytical mechanics. Publication of *Mécanique Analytique* (1788).

1777–1859 Lewis Poinsot: Publication of theory of couples in *Éléments de Statique* (1803).

1800–1860 Principle of conservation of energy—the result of the combined efforts of many workers.

1853 Publication of *Elements of Analytical Mechanics* by W. H. C. Bartlett—first Mechanics text for technical students by an American author.

1860– —— Refinement of technique and method of presentation.

APPENDIX B

Chronologically Important Dates of Mechanics of Materials

1638 First important discussion of Strength of Materials by Galileo.
1660 Robert Hooke discovers his theory of springs.
1680 Mariotte discovers elongation and compression of beam fibers.
1694 James Bernoulli discovers the equation of the elastic curve of a beam.
1729 First crude testing machine using steelyard; Pieter Musschenbroek.
1747 L'École des Ponts et Chaussées started by Perronnet.
1757 Leonhard Euler publishes his column formula.
1773 Charles Augustin Coulomb establishes position of neutral axis; presents concepts of internal resisting moment and of shear deformation.
1775 L'École des Ponts et Chaussées given legal recognition.
1787 Coulomb publishes the first discussion on torsion.
1807 Thomas Young's *Lectures on Natural Philosophy*, containing Young's definition of Modulus of Elasticity, published. Young was first to treat shear as an elastic strain. The beginning of practical elasticity.
1817 United States Military Academy began teaching applied science.
1821 Navier writes the general equations of elasticity.
1822 Cauchy presents the origin of the theory of stress.
1822 Gardiner Lyceum starts the teaching of practical science in America.
1823 Brewster does the first work on photo-elasticity.
1826 Navier publishes the first good text on Mechanics of Materials. Flexure formula in present form.
1827 P. Lagerhjelm of Sweden built first testing machine with a hydraulic press and balanced lever.
1827 Cauchy makes the first attempt to apply general equations of elasticity to practical problems.
1827 Cauchy uses two elastic constants for first time.
1828 Poisson also develops the general equations of elasticity and the Poisson ratio.

1832 First testing machine in United States at Franklin Institute.
1835 Rensselaer Polytechnic Institute grants first degree of Civil Engineer in English-speaking world.
1838 Sir William Fairbairn makes first study of riveted joints.
1842 Wertheim defines elastic limit as now defined.
1847 Squire Whipple gives solution of jointed truss.
1852 Lamé starts swing to multi-constancy.
1855 Saint-Venant presents his solution of the torsion problem.
1856 Saint-Venant publishes his work on flexure.
1857 Clapeyron's three-moment theorem presented.
1858 Solution for statically-indeterminate structures by L. F. Menabrea.
1860 Luders' lines discovered.
1862 Morrill Land-Grant Act passed.
1864 Graphical statics revived by C. Culmann.
1864 Maxwell invented Maxwell-Cremona stress diagram.
1864 Solution for statically-indeterminate structures by C. Maxwell.
1865 Method of sections by August Ritter.
1867 The influence line by E. Winkler.
1868 Otto Mohr develops a graphical solution for beam deflections.
1870 Saint-Venant publishes first work on plastic flow of metals.
1871 DeVolson Wood publishes first American text on Strength of Materials.
1873 Area-moment method by Charles E. Greene.
1875 Castigliano's Theorem.
1882 Otto Mohr develops Mohr diagram for stress at point.
1886 H. Müller-Breslau gives method for statically-indeterminate structures.
1903 Soap-film method of stress analysis by L. Prandtl.
1920 Photo-elastic stress analysis becomes practical method.
1922 The awakening of the American engineering profession.
1924 First International Congress of Applied Mechanics held at Delft, Netherlands.
1925 Solution for statically indeterminate structures by A. Ostenfeld.
1930 First publication on plasticity written in United States.
1932 Hardy Cross' *Analysis of Continuous Frames.*
1932 Stress analysis by use of brittle lacquers.
1935 Photo-elastic stress analysis enters the three-dimensional field.

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